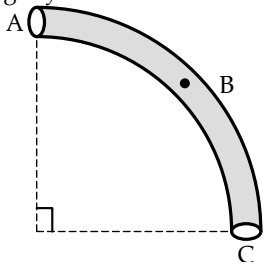
MOCK TEST PAPER 2

General Instructions: *Same as Mock Test Paper 1.*

Physics

Section A

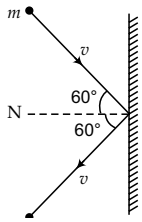
Q. 1. The tube AC forms a quarter circle in a vertical plane. Ball B has a cross-sectional area slightly smaller than that of the tube and can move through it without friction. B is placed at A and displaced slightly. It will:



(1) always be in contact with the inner wall of the tube
(2) always be in contact with the outer wall of the tube
(3) initially be in contact with the inner wall and later with the outer wall of the tube
(4) initially be in contact with the outer wall and later with the inner wall of the tube

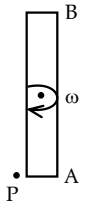
Q. 2. A galvanometer of 50 ohms resistance has 25 divisions. A current of 4×10^{-4} ampere gives a deflection of one division. To convert this galvanometer into a voltmeter with a range of 25 volts, it should be connected with a resistance of:
(1) 2500 Ω as a shunt **(2)** 2450 Ω as a shunt
(3) 2550 Ω in series **(4)** 2450 Ω in series

Q. 3. A rigid ball of mass m strikes a rigid wall at 60° and gets reflected without loss of speed, as shown in the figure below. The value of impulse imparted by the wall on the ball will be:



(1) $\frac{mv}{2}$ **(2)** $\frac{mv}{3}$ **(3)** mv **(4)** $2mv$

Q. 4. A rod of mass 'M' and length 'L', lying on a frictionless horizontal surface, is initially given an angular velocity 'ω' about the vertical axis with it's centre of mass at rest but it's circular motion is not fixed. Subsequently, end A of the rod collides with nail P, which is near A, in such a way that end A becomes stationary immediately after impact. The velocity of end B just after collision will be:



(1) ωL **(2)** $\frac{\omega L}{2}$ **(3)** $\frac{\omega L}{4}$ **(4)** $\frac{7\omega L}{3}$

Q. 5. If R is the radius of the earth and g the acceleration due to gravity on the earth's surface, the mean density of the earth is:
(1) $\frac{4\pi G}{3gR}$ **(2)** $\frac{3\pi R}{4gG}$ **(3)** $\frac{3g}{4\pi RG}$ **(4)** $\frac{\pi Rg}{12G}$

Q. 6. A particle is oscillating according to the equation $x = 7 \cos 0.5\pi t$, where t is in seconds. The point moves from the position of equilibrium to maximum displacement in time:
(1) 4.0 second **(2)** 2.0 second
(3) 1.0 second **(4)** 0.5 second

Q. 7. A ball falling in a lake of 200 m shows a decrease of 0.1 % in its volume at the base of the lake. The bulk modulus of elasticity of the material of the ball is (take $g = 10$ m/s^2):
(1) 10^9 N/m^2 **(2)** 2×10^9 N/m^2
(3) 3×10^9 N/m^2 **(4)** 4×10^9 N/m^2

Q. 8. In a capillary tube, water rises to a height of 4 cm. If the cross-sectional area of the tube were one-fourth, water would have risen to a height of:
(1) 2 cm **(2)** 4 cm **(3)** 8 cm **(4)** 16 cm

Q. 9. Given below are two statements:

Statement I: The brightness of a bulb connected in series with a coil decreases when an iron rod is inserted into the coil.

Statement II: Inserting an iron rod into a coil increases the circuit's inductance, reducing the current flow and subsequently dimming the bulb.

Choose the correct answer from the options given below:

(1) Statement I is true, Statement II is true and Statement II is the correct explanation of Statement I.

(2) Statement I is true, Statement II is true, but Statement II is not the correct explanation of Statement I.

(3) Statement I is true, Statement II is false.

(4) Statement I is false, Statement II is true.

Q. 10. A solid cone of height 25 cm and base diameter 25 cm floats in water with its vertex downwards such that 20 cm of its axis is immersed. The additional weight that must be placed on the centre of the base such that the cone now is completely immersed in water is:

(1) 1 kg **(2)** 2 kg

(3) 3 kg **(4)** 4 kg

Q. 11. The electric field in a certain region is given by $\vec{E} = (5\hat{i} - 3\hat{j})$ kV/m. The potential difference $V_B - V_A$ between points A and B, having coordinates (4, 0, 3)m and (10, 3, 0)m respectively, is equal to

(1) 21 kV **(2)** –21 kV **(3)** 39 kV **(4)** –39 kV

Q. 12. Two monoatomic ideal gases 1 and 2 of molecular masses M_1 and M_2 are enclosed in separate containers kept at the same temperature. The ratio of the speed of sound in gas 1 to that in gas 2 is:

(1) $\sqrt{\frac{M_1}{M_2}}$ **(2)** $\sqrt{\frac{M_2}{M_1}}$ **(3)** $\frac{M_1}{M_2}$ **(4)** $\frac{M_2}{M_1}$

Q. 13. The electric field of an electromagnetic wave in free space is represented as $E_0\cos(\omega t - kz)\hat{i}$. The corresponding magnetic induction vector will be:

(1) $\vec{B} = E_0 c\cos(\omega t - kz)\hat{j}$

(2) $\vec{B} = \frac{E_0}{c}\cos(\omega t - kz)\hat{j}$

(3) $\vec{B} = E_0 c\cos(\omega t + kz)\hat{j}$

(4) $\vec{B} = \frac{E_0}{c}\cos(\omega t + kz)\hat{j}$

Q. 14. The wavefront of a light beam is given by the equation $x + 2y + 3z = C$ (where C is arbitrary constant), then the angle made by the direction of light with the y-axis is:

(1) $\cos^{-1}\frac{1}{\sqrt{14}}$ **(2)** $\sin^{-1}\frac{2}{\sqrt{14}}$

(3) $\cos^{-1}\frac{2}{\sqrt{14}}$ **(4)** $\sin^{-1}\frac{3}{\sqrt{14}}$

Q. 15. A film projector magnifies a 100 cm^2 film strip on a screen in such a way that the distance between the screen and the projector is divided in the ratio of 2:1 by the lens. Then the area of magnified film on the screen is:

(1) 1600 cm^2 **(2)** 400 cm^2

(3) 800 cm^2 **(4)** 200 cm^2

Q. 16. What is the number of wavelengths that can be emitted by hydrogen atoms when an electron falls from the fifth orbit to its ground state?

(1) 4 **(2)** 5 **(3)** 10 **(4)** 3

Q. 17. If the short series limit of the Balmer series for hydrogen is 3646 Å, calculate the atomic number of the element which gives X-ray wavelength down to 1.0 Å. Identify the element:

(1) $Z = 21$ **(2)** $Z = 31$

(3) $Z = 11$ **(4)** $Z = 5$

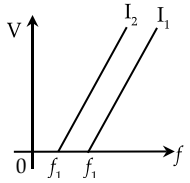
Q. 18. The wavelength of a neutron with energy 1 eV is closest to:

(1) 10^{-2} cm **(2)** 10^{-4} cm

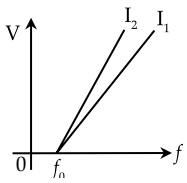
(3) 10^{-6} cm **(4)** 10^{-8} cm

Q. 19. A photoelectric experiment is performed at two different light intensities I_1 and I_2 ($>I_1$). Choose the correct graph showing the variation of stopping potential versus frequency of light.

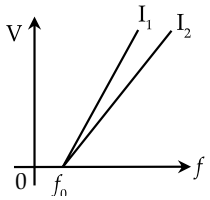
(1)



(2)



(3)



(4) None of these

Q. 20. If a semiconductor with an intrinsic carrier concentration of 1.41×10^{16} m^{-3} is doped with 10^{21} m^{-3} phosphorus, then the concentration of holes at room temperature will be:

(1) 2×10^{21} **(2)** 2×10^{11}

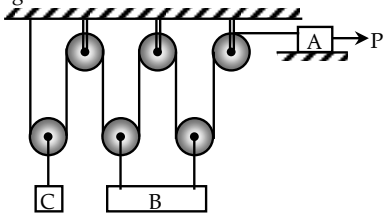
(3) 1.41×10^{10} **(4)** 1.41×10^{16}

Section B

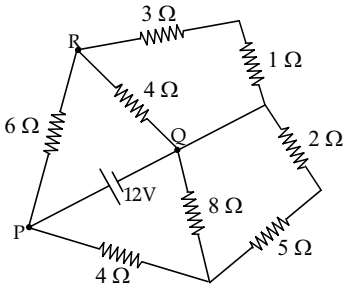
Q. 21. A ray of light passes through an equilateral prism (μ=1.5) such that $i = e$ and $e = \frac{3}{4}$ of angle of the prism. The angle of deviation is:

Q. 22. An Ideal Gas at 27°C is compressed adiabatically to $\frac{8}{27}$ of its original volume. The rise in temperature is (take γ =5/3):

Q. 23. Three blocks A, B and C, each with mass m, are arranged in a pulley mass system as shown. The coefficient of friction between Block A and the horizontal surface is equal to 0.5 and a Force P acts on 'A' in the direction shown. The value of P/mg so that Block 'C' doesn't move is.....



Q. 24. Power dissipated by the circuit is ..... W.



Q. 25. There is a constant homogeneous electric field of 100 Vm^{-1} within the region $x = 0$ and $x = 0.167$ m pointing in the positive x-direction. There is a constant homogeneous magnetic field B within the region $x = 0.167$ m and $x = 0.334$ m pointing in the z-direction. A proton at rest at the origin ($x = 0, y = 0$) is released in the positive x-direction. The minimum strength of the magnetic field B, so that the proton will come back to $x = 0$, $y = 0.167$ m (mass of the proton = 1.67×10^{-27} kg) is.....mT.

Chemistry

Section A

Q. 26. Hex–4–ene–2–ol on treatment with PCC gives 'A'. 'A' on reaction with sodium hypoiodite gives 'B', which on further heating with soda lime gives 'C'. The compound 'C' is:

(1) 2–pentene **(2)** propionaldehyde
(3) 2–butene **(4)** 4–methylpent–2–ene

Q. 27. Which of the following pairs of species have the same bond order?

(1) N_2, NO^+ **(2)** O_2, NO^+
(3) N_2, O_2^- **(4)** CO, NO

Q. 28. The K_{sp} for bismuth sulphide (Bi_2S_3) is 1.08×10^{-73}. The solubility of Bi_2S_3 in mol L^{-1} at 298 K is:

(1) 1.0×10^{-15} **(2)** 2.7×10^{-12}
(3) 3.2×10^{-10} **(4)** 4.2×10^{-8}

Q. 29. The element Z = 107 and Z = 109 have been made recently; element Z = 108 has not yet been made. Indicate the group in which you will place the above elements.

(1) 7, 8, 9 **(2)** 5, 6, 7
(3) 8, 9, 10 **(4)** 4, 5, 6

Q. 30. K_{a_1}, K_{a_2} and K_{a_3} are the respective ionisation constants for the following reactions (a), (b) and (c).

(a) $H_2C_2O_4 \rightleftharpoons H^+ + HC_2O_4^-$

(b) $HC_2O_4^- \rightleftharpoons H^+ + C_2O_4^{2-}$

(c) $H_2C_2O_4 \rightleftharpoons 2H^+ + C_2O_4^{2-}$

The relationship between K_{a_1}, K_{a_2}, and K_{a_3} is given as

(1) $K_{a_3} = K_{a_1} + K_{a_2}$ **(2)** $K_{a_3} = K_{a_1} - K_{a_2}$
(3) $K_{a_3} = K_{a_1} / K_{a_2}$ **(4)** $K_{a_3} = K_{a_1} \times K_{a_2}$

Q. 31. Enthalpy of fusion of a liquid is 1.435 kcal mol^{-1} and molar entropy change is 5.26 cal $mol^{-1}K^{-1}$. Hence melting point of liquid is:

(1) 100°C **(2)** 0°C
(3) 373°C **(4)** –273°C

Q. 32. In $Fe_4\left[Fe(CN)_6\right]_3$ the oxidation state of the complexed iron is:

(1) +3 **(2)** +2 **(3)** +4 **(4)** +6

Q. 33. Assertion A and the other is labelled as Reason R:

Assertion A: O_2^- is paramagnetic.

Reason R: O_2^- have all paired electrons.

In the light of the above statements, choose the correct answer from the options given below:

(1) Both A and R are true and R is NOT the correct explanation of A.
(2) A is true but R is false.
(3) A is false but R is true.
(4) Both A and R are true and R is the correct explanation of A.

Q. 34. Given below are two statements:

Statement I: The ionisation energy of Cu is more than that of K though both have a 4s configuration.

Statement II: The 18 electron shell of Cu shields nuclear charge more effectively than the shell of K.

Choose the correct answer from the options given below:

(1) Statement I is incorrect but statement II is true.

(2) Both statements I and II are true.

(3) Both statements I and statement II are false.

(4) Statement I is correct but statement II is false.

Q. 35. Ethyl methyl vinyl amine has the structure:

(1) $CH_3CH_2–N(CH_3)–CH_2CH=CH_2$

(2) $CH_3CH_2–N(CH_3)–CH=CH_2$

(3) $CH_2=CH–N(CH_3)–CH=CH_2$

(4) $CH_3–N(CH_3)–CH=CH_2$

Q. 36. The correct name of $(CO)_3Fe–CO–Fe(CO)_3$ is:

(1) Tri-μ-carbonyl bis (tricarbonyl iron (0))

(2) Hexacarbonyl iron (III) μ-tricarbonyl ferrate (0)

(3) Tricarbonyl iron (0) μ-tricarbonyl iron (0) tricarbonyl

(4) Nonacarbonyl iron

Q. 37. Given below are two statements:

Statement I: Complexes of type MA_5B and MA_6 do not show geometrical isomerism.

Statement II: Coordination compounds having coordination number 6 do not exhibit geometrical isomerism.

Choose the correct answer from the options given below:

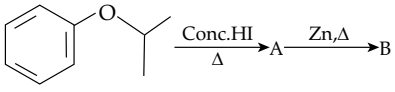
(1) Statement I is incorrect but statement II is true.

(2) Both statement I and statement II are true.

(3) Both statement I and statement II are false.

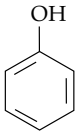
(4) Statement I is correct but statement II is false.

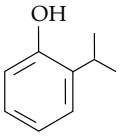
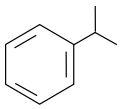
Q. 38. Compound I is heated with conc. HI to give a hydroxy compound A which is further heated with Zn dust to give compound B. Identify A and B.

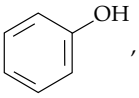


I

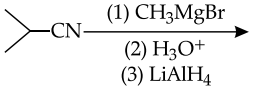
(1) A =  , B = 

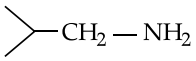
(2) A =  , B =

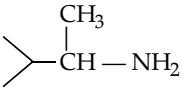
(3) A =  , B = 

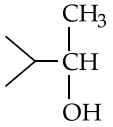
(4) A =  , B = 

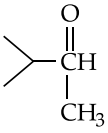
Q. 39. The major product of the following reaction is:



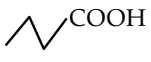
(1) 

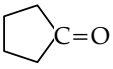
(2) 

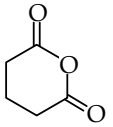
(3) 

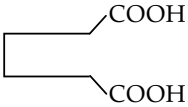
(4) 

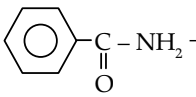
Q. 40. The product formed when adipic acid is heated:

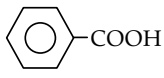
(1) 

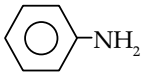
(2) 

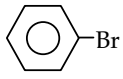
(3) 

(4) 

Q. 41.  product (A) is:

(1) 

(2) 

(3) 

(4) None the above

Q. 42. $(C_6H_{12}O_5)_n + nH_2O \xrightarrow{(X)} nC_{12}H_{22}O_{11}$

$C_{12}H_{22}O_{11} + H_2O \xrightarrow{(Y)} 2C_6H_{12}O_6$

Enzymes X and Y are:

(1) Zymase and Invertase

(2) Diatase and Maltase

(3) Maltase and Diatase

(4) Diatase and Zymase

Q. 43. Given below are two statements:

Statement I: At a frequency $v > v_0$, the ejected e^- come out with certain kinetic energy.

v = incident frequency; v_0 = threshold frequency

Statement II: The kinetic energy of these electrons increase with the increase of frequency of the light used.

(1) Statement I is correct and statement II is incorrect.

(2) Statement I is incorrect and statement II is correct.

(3) Both statement I and II are incorrect.

(4) Both statement I and II are correct.

Q. 44. In the Hunsdiecker reaction:

(1) number of carbon atoms decrease

(2) number of carbon atoms increase

(3) number of carbon atoms remain same

(4) none of the above

Q. 45. In the reaction sequence,

$$CaC_2 \xrightarrow{H_2O} A \xrightarrow[Hg^{+2}]{dil.H_2SO_4} B \xrightarrow[Ni]{H_2} C,$$

the product C is:

(1) CH_3OH **(2)** CH_3CHO

(3) C_2H_5OH **(4)** C_2H_4

Section B

Q. 46. Given that the temperature coefficient for the saponification of ethyl acetate by NaOH is 1.75. The activation energy for the saponification of ethyl acetate is ..... kcal mol^{-1}.

Q. 47. Cyclohexane-1,4-dione is a polar compound, having dipole moment value of 1.2 D. If mol fraction of its chair form is 0.80, the dipole moment of twisted boat form will be .....

Q. 48. The resistance of a conductivity cell containing 0.01 M KCl solution at 298 K is 1750Ω. If the conductivity of 0.01 M KCl solution at 298 K is 0.152×10^{-3} S cm^{-1}, then the cell constant of the conductivity cell is _____ $\times 10^{-3}$ cm^{-1}.

Q. 49. An organic liquid, A, is immiscible with water. When boiled together with water, the boiling point is 90°C at which the partial vapour pressure of water is 526 mm Hg. The atmospheric pressure is 736 mm Hg. The weight ratio of the liquid and water collected is 2.5: 1. The molecular weight of the liquid is ..... g.

Q. 50. The quantity of benzene, when 91.2 g of Phenylmagnesium iodide is treated with 4.2 g of Pent-4-yn-1-ol at STP would be produced ..... *l*.

Mathematics

Section A

Q. 51. The remainder when $32^{32^{32}}$ is divided by 7 is m. The value of limit $\lim\limits_{x\to 0}\left(\frac{\sin x}{x}\right)^{\frac{\sin x}{x-\sin x}}$ is e^n. The quadratic equation whose roots are $\frac{1}{m}$ and $\frac{1}{n}$ is:

(1) $x^2 - 3x - 4 = 0$ **(2)** $4x^2 + 3x - 1 = 0$

(3) $4x^2 - 3x + 1 = 0$ **(4)** $x^2 + 3x - 4 = 0$

Q. 52. $\int_0^1 \frac{dx}{(x^2-2x+2)^3} =$

(1) $\frac{3\pi+8}{32}$ **(2)** $\frac{\pi+1}{4}$

(3) 0 **(4)** $\frac{2\pi}{3}$

Q. 53. Consider system of linear equations: $x + y + z = 6$; $x + 2y + 3z = 10$; $x + 2y + k_1z = k_2$. Then, which one of the following is correct:

(1) Unique solution for $k_1 \neq 2$

(2) No solution for $k_1 \neq 3$ and $k_2 = 10$

(3) Infinite no. of solution for $k_1 = 3$ and $k_2 = 10$

(4) None of the above

Q. 54. If the solution of the differential equation $\frac{dy}{dx}(x^2y^3 + xy) = 1$ is $x(y^m + n + c_1e^{-y^2/k}) = l$, then ${}^{m-n+k+l}C_m$ is equal to (where c is constant of integration):

(1) 10 **(2)** 5

(3) 12 **(4)** Not defined

Q. 55. A triangle has two of its sides along the axis, its third side touches the circle

$x^2 + y^2 - 2\lambda x - 2\lambda y + \lambda^2 = 0$.

The equation of locus of the circumcentre of the triangle is $m(x+y) - \lambda = \frac{nxy}{\lambda}$, then $(m + n)$ is equal to:

(1) 4 **(2)** 3 **(3)** 0 **(4)** 6

Q. 56. Let PQR be a triangle. Points A, B and C are taken on the sides PQ, QR and RP respectively, such that $\frac{PA}{PQ} = \frac{QB}{QR} = \frac{RC}{RP} = m$. The value of m for which area of the triangle formed by $\vec{PB}$, $\vec{QC}$ and $\vec{RA}$ is the least is:

(1) $\frac{1}{3}$ **(2)** $\frac{1}{4}$ **(3)** $\frac{1}{2}$ **(4)** $\frac{1}{6}$

Q. 57. The root of the equation

$$[x\ 1\ 2]\begin{bmatrix}0 & 1 & 1\\ 1 & 0 & 1\\ 1 & 1 & 0\end{bmatrix}\begin{bmatrix}x\\ -1\\ 1\end{bmatrix} = 0 \text{ is .....}$$

(1) $\frac{1}{3}$ **(2)** $-\frac{1}{3}$ **(3)** 0 **(4)** 1

Q. 58. Consider points $P\left(\sqrt{13}, 0\right)$ and $Q\left(2\sqrt{13}, 0\right)$ lying on x-axis. These points are rotated in an anticlockwise direction about the origin through an angle $\tan^{-1}\left(\frac{2}{3}\right)$. Let the new position of P and Q be P′ and Q′ respectively. If the foot of perpendicular from (4, – 2) to line P′Q′ is (α, β), then 13 (α + β) is equal to:
(1) 30 **(2)** 40 **(3)** 45 **(4)** 25

Q. 59. From a pack of well shuffled cards, one card is drawn randomly. A gambler bets that it is either a diamond or a king. The odds in favour of his winning the bet are:
(1) 9:4 **(2)** 4:9 **(3)** 5:7 **(4)** 9:7

Q. 60. Let $z_1 = 10 + 6i$, $z_2 = 4 + 6i$ and z is a complex number such that $\arg\left(\frac{z-z_1}{z-z_2}\right)=\frac{\pi}{2}$, then the value of $|z - 7 - 9i|$ is..... .
(1) 2 **(2)** 6 **(3)** $3\sqrt{2}$ **(4)** 8

Q. 61. PQR is a triangle where P = (2, 3, 5), Q = (– 1, 2, 2) and R = (α, 5, β). If the median through P is equally inclined to the axis, then ${}^nC_0 + 3{}^nC_1 + 5{}^nC_2 +$..... + $(2n + 1)\, C_n$ is equal to, where $n = \alpha + \beta$:
(1) 2^{15} **(2)** 2^{18} **(3)** 2^{20} **(4)** 2^{19}

Q. 62. If $g : R \to \left(0, \frac{\pi}{2}\right]$, then the value of k for which $g(x) = \cot^{-1}(x^2 + x + k)$ is an onto function, is given by:
(1) $\frac{1}{3}$ **(2)** $\frac{1}{4}$ **(3)** $\frac{1}{8}$ **(4)** $\frac{1}{2}$

Q. 63. If a, b, c are in AP; a, b, d are in GP; then $a, a - b, d - c$ are in
(1) AP **(2)** GP
(3) HP **(4)** AP and GP both

Q. 64. $\frac{1}{1!\cdot(n-1)!}+\frac{1}{3!\cdot(n-3)!}+\frac{1}{5!\cdot(n-5)!}+\ldots$ is equal to:
(1) $\frac{2^{n-1}}{n!}$ for even values of n only
(2) $\frac{2^{n-1}+1}{n!}$ –1 for odd values of n only
(3) $\frac{2^{n-1}}{n!}$ for all $n \in N$
(4) $\frac{2^n}{(n-1)!}$ for all $n \in N$

Q. 65. If $\int\frac{\cos^2 x+\sin 2x}{(2\cos x-\sin x)^2}dx=\frac{1}{f(x)}-\frac{m}{5}\log_e f(x)+\frac{n}{5}\log_e(g(x))-\frac{r}{5}h(x)+c$; where c is constant of integration and $m > n$, then which of the following is correct?
(1) $f(0) + g(0) + h(0) = 4$
(2) $m + n + r = 3$
(3) Minimum value of $g(x) = – 2$
(4) $f'(0) + g'(0) + h'(0) = 0$

Q. 66. Let U = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11} and B = {1, 3, 4, 5, 6}. Then, the number of subsets of B^C, which must contain 10, is:
(1) 16 **(2)** 32 **(3)** 64 **(4)** 8

Q. 67. The equation of the parabola whose focus is (1, 1) and tangent at the vertex is $x + y = 1$, is
(1) $x^2 + y^2 – 2xy – 4x – 4y + 4 = 0$
(2) $x^2 + y^2 – 2xy + 4x + 4y + 4 = 0$
(3) $x^2 + y^2 – 2xy – 4x – 4y – 4 = 0$
(4) $x^2 + y^2 – 2xy + 4x + 4y – 4 = 0$

Q. 68. The domain of the function $f(x)=\frac{1}{\sqrt{x-2\{x\}}}+\ln\{x\}$ (where { } denotes fractional part function) is
(1) R – I **(2)** (1, ∞)
(3) [1, ∞) – I^+ **(4)** (0, ∞)

Q. 69. If the hyperbolas, $x^2 + 3xy + 2y^2 + 2x + 3y + 2 = 0$ and $x^2 + 3xy + 2y^2 + 2x + 3y + c = 0$ are conjugate of each other, then the value of 'c' is equal to:
(1) – 2 **(2)** 4 **(3)** 0 **(4)** 1

Q. 70. Five numbers x_1, x_2, x_3, x_4, x_5 are randomly selected from the numbers 1, 2, 3,…, 18 and are arranged in the increasing order ($x_1 < x_2 < x_3 < x_4 < x_5$). The probability that $x_2 = 7$ and $x_4 = 11$ is:
(1) $\frac{1}{136}$ **(2)** $\frac{1}{72}$ **(3)** $\frac{1}{68}$ **(4)** $\frac{1}{34}$

Section B

Q. 71. If $\sin^2(10°)\sin(20°)\sin(40°)\sin(50°)\sin(70°) = \alpha - \frac{1}{16}\sin(10°)$, then $16 + \alpha^{-1}$ is equal to _____.

Q. 72. If $\frac{f(x)}{\sin^2 x}=-\cos^{-1}\left(\frac{2\sqrt{2}x}{\pi}\right)-|f(x)|$, then the value of $\frac{f(\pi/4)}{(-\pi/32)}$ is ..... .

Q. 73. The value of $\lim\limits_{x\to 2}\frac{2^x+2^{3-x}-6}{\sqrt{2^{-x}}-2^{1-x}}$ is ..... .

Q. 74. If complex number z_1 and z_2 satisfy the equations $|z_1| = 6$ and $|z_2 – 4| + |z_2 + 4| = 10$, then the maximum value of $|z_1 – z_2|$ is ..... .

Q. 75. If $\ln\left((e-1)e^{xy}+x^2\right)=x^2+y^2$, then $\left(\frac{dy}{dx}\right)_{(1,0)}$ is equal to ..... .

Answers

Physics

Q. No.	Answer	Topic's name	Q. No.	Answer	Topic's name
1	(3)	Circular Motion	14	(3)	Wave Optics
2	(4)	Conversion of Galvanometer into Voltmeter	15	(1)	Optical instruments
3	(3)	Impulse	16	(3)	Atoms
4	(3)	Rotational Motion	17	(2)	Spectral Series
5	(3)	Gravitation	18	(4)	Matter Wave
6	(3)	Simple Harmonic Motion	19	(4)	Photoelectric Effect
7	(2)	Bulk Modulus of Elasticity	20	(4)	Semiconductor Devices
8	(3)	Capillary rise	21	[30]	Refraction through Prism
9	(1)	Inductance and AC Circuit	22	[375]	Adiabatic Process
10	(2)	Buoyancy and Archimedes' principle	23	[5.00]	Mass-Pulley System
11	(2)	Relation between Electric Field and Potential	24	[36.00]	Power in Electrical Circuits
12	(2)	Speed of Sound	25	[7.07]	Magnetic Effects of Current
13	(2)	Electromagnetic Waves			

Chemistry

Q. No.	Answer	Topic's name	Q. No.	Answer	Topic's name
26	(3)	Mix reaction	39	(3)	Carbonyl Compound
27	(1)	Chemical Bonding	40	(2)	Carboxylic Acid
28	(1)	Ionic Equilibrium	41	(2)	Nitrogen Compound
29	(1)	Periodic Table	42	(2)	Biomolecules
30	(4)	Chemical Equilibrium	43	(4)	Structure of atom
31	(2)	Chemical Energetics	44	(1)	Halogen Derivatives
32	(2)	Redox Reaction	45	(3)	Alcohol, Ether and Phenol
33	(2)	Chemical Bonding	46	[10.20]	Chemical Kinetics
34	(4)	d and f block	47	[6.0]	Dipole Moment
35	(2)	IUPAC	48	[266]	Electrochemistry
36	(1)	Coordination Compound	49	[112.70]	Solution
37	(4)	Coordination Compound	50	[1.12]	Aromatic Compound
38	(4)	Chemical Reactions of Ethers			

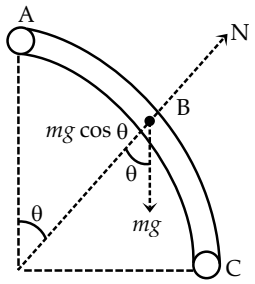
Mathematics

Q. No.	Answer	Topic's name	Q. No.	Answer	Topic's name
51	(2)	Binomial theorem	64	(3)	Binomial Theorem
52	(1)	Definite Integration	65	(4)	Integration
53	(3)	Determinants	66	(2)	Sets
54	(1)	Differential Equations	67	(1)	Parabola
55	(1)	Sraight Lines	68	(3)	Function
56	(3)	Vectors	69	(3)	Hyperbola
57	(1)	Matrices	70	(3)	Probability
58	(2)	Straight Lines	71	[80.00]	Trigonometric Identities
59	(2)	Probability	72	[8.00]	Inverse Trigonometric Functions
60	(3)	Complex Numbers	73	[8.00]	Limits
61	(4)	Straight Lines	74	[11.00]	Complex Number
62	(2)	Functions	75	[2.00]	Differentiation
63	(2)	Sequence and Series			

ANSWERS WITH EXPLANATION

Physics

1. Option (3) is correct



For circular motion of ball in the tube,

$$mg\cos\theta - N = \frac{mv^2}{r}$$

As Ball B moves down, the value of θ increases, cos θ decreases.

$$N = mg\cos\theta - \frac{mv^2}{r}$$

So, N continuously decreases and it becomes 0 when $\cos\theta = \frac{v^2}{rg}$. Thereafter, it becomes negative because the ball leaves contact with surface A and comes in contact with outer part C.

[for e.g., at lowest point we can say,

$$N = mg\cos 90° - \frac{mv^2}{r}$$

$$= -\frac{mv^2}{r} \rightarrow \text{contact with C]}$$

2. Option (4) is correct.

According to the question, $V = I_g(R + R_g)$.

Full scale deflection current of galvanometer:

$$I_g = 25 \times 4 \times 10^{-4}$$
$$= 100 \times 10^{-4}\ A$$
$$= 10^{-2}\ A$$

As $$R = \frac{V}{I_g} - R_g$$

$$= 25 \times 10^2 - 50$$
$$= 2450\ \Omega \text{ in series}$$

3. Option (3) is correct.

Initial momentum of the ball perpendicular to the wall,

$$P_i = m(-v\cos 60°)$$

$$P_i = \frac{-mv}{2}$$

Similarly, Final momentum of the ball perpendicular to the wall,

$$P_f = m(v\cos 60°)$$

$$P_f = \frac{mv}{2}$$

Now, Impulse = Change in momentum

$$I = P_f - P_i$$

$$I = \frac{mv}{2} - \left(\frac{-mv}{2}\right)$$

$$I = \frac{mv}{2} + \frac{mv}{2}$$

$$I = mv$$

4. Option (3) is correct.

Applying conservation of angular momentum about P

$$\Rightarrow \quad \frac{ML^2}{12}\omega = \frac{ML^2}{3}\omega_1$$

$$\Rightarrow \quad \omega_1 = \frac{\omega}{4}$$

$$\therefore \quad v_B = \frac{\omega L}{4}$$

5. Option (3) is correct.

$$g = \frac{GM}{R^2}$$

(Given R → Radius of Earth,
ρ → Mean density of Earth)

$$g = \frac{G\left(\frac{4}{3}\pi R^3\rho\right)}{R^2};$$

$$g = \frac{4}{3}\pi R\,\rho G$$

$$\rho = \frac{3}{4}\cdot\frac{g}{\pi RG}$$

6. Option (3) is correct.

Given $x = 7\cos 0.5\,\pi t$

General form of equation

$$x = A\cos\omega t$$

Given equation compare with standard form

$$\omega = \frac{2\pi}{T} = 0.5\,\pi;\ T = 4\ s$$

$$T' = \frac{T}{4} = \frac{4}{4} = 1\ s$$

7. Option (2) is correct.

$$\Delta P = h\rho g$$
$$= 200 \times 10^3 \times 10$$
$$= 2 \times 10^6\ N/m^2$$

$$K = \frac{\Delta P}{\frac{\Delta V}{V}} = \frac{2\times10^6}{\frac{1}{100}}$$

$$= 2\times 10^9 \text{ N/m}^2$$

8. Option (3) is correct.

$$h = \frac{2T\cos\theta}{r\rho g}$$

$$r_1 h_1 = r_2 h_2$$

and $$A = 2\pi r^2$$

$$\Rightarrow \quad r \propto \sqrt{A}$$

$$\therefore \quad \sqrt{A_1}\ h_1 = \sqrt{A_2}\ h_2$$

$$\Rightarrow \quad \sqrt{A} \times 4 = \sqrt{\frac{A}{4}} \times h_2$$

$$\Rightarrow \quad h_2 = 8 \text{ cm}$$

9. Option (1) is correct.

The insertion of the iron rod increases the inductance of the circuit, which in turn reduces the current flow and subsequently dims the bulb. Thus, Statement II provides a valid explanation for the observed effect described in Statement I.

10. Option (2) is correct.

Height of cone = 25 cm

Base diameter = 25 cm

Volume of the cone $= \frac{1}{3}\pi r^2 h$

$$= \frac{1}{3} \times 3.14 \times (12.5)^2 \times 25 \text{ cm}^3$$

$$= 4088 \text{ cm}^3$$

The volume of already immersed part of the cone

$$V' = \frac{1}{3} \times 3.14 \times (10)^2 \times 20$$

$$= 2093 \text{ cm}^3$$

Water to be displaced for complete immersion

$$= 4088 - 2093$$

$$= 1995 \text{ g}$$

$$\cong 2 \text{ kg}$$

Additional mass to be put

$$= 2000 \text{ g} = 2 \text{ kg}$$

11. Option (2) is correct.

Here, $\vec{E} = 5\hat{i} - 3\hat{j}$ kV/m

$$V_B - V_A = -\int_{r_A}^{r_B} \vec{E}.dr$$

$$= -\int_{(4,0,3)}^{(10,3,0)} \left(5\hat{i} - 3\hat{j}\right).\left(dx\hat{i} + dy\hat{j} + dz\hat{k}\right)$$

$$= -\int_4^{10} 5dx - \int_0^3 (-3)dy + 0 = -5\left[x\right]_4^{10} + 3\left[7\right]_0^3$$

$$= -5(10-4) + 3(3-0) = -30 + 9 = -21 \text{ kV}$$

12. Option (2) is correct.

As $$v = \sqrt{\frac{\gamma RT}{M}}$$

$$\therefore \quad \frac{v_1}{v_2} = \sqrt{\frac{M_2}{M_1}}$$

13. Option (2) is correct.

$$E_0 = B_0 c$$

$$\Rightarrow \quad B_0 = \frac{E_0}{c}$$

$$\therefore \quad B = \frac{E_0}{c}\cos(\omega t - kz)\hat{j}$$

14. Option (3) is correct.

Here direction of light is given by normal vector

$$\hat{n} = \hat{i} + 2\hat{j} + 3\hat{k}$$

$\therefore$ angle made by the $\hat{n}$ with y-axis is given by

$$\cos\beta = \frac{2}{\sqrt{1^2+2^2+3^2}} = \frac{2}{\sqrt{14}}$$

$$\beta = \cos^{-1}\frac{2}{\sqrt{14}}$$

15. Option (1) is correct.

$$A_I = m^2 A_0$$

$$\therefore \quad A_I = (4)^2 \times 100$$

$$\therefore \quad A_I = 1600 \text{ cm}^2$$

16. Option (3) is correct.

$$N = \frac{n(n-1)}{2} = \frac{5(5-1)}{2} = 10$$

17. Option (2) is correct.

The short limit of the Balmer series is given by

$$\bar{\nu} = \frac{1}{\lambda}$$

$$= R\left(\frac{1}{2^2} - \frac{1}{\infty^2}\right) = \frac{R}{4}$$

$$\therefore \quad R = \frac{4}{\lambda} = \frac{4}{3646} \times 10^{10} \text{ m}^{-1}$$

Further, the wavelengths of the Kα series are given by the relation:

$$\bar{\nu} = \frac{1}{\lambda}$$

$$= R(Z-1)^2\left(\frac{1}{1^2} - \frac{1}{n^2}\right)$$

The maximum wave number correspnods to $n = \infty$ and therefore, we must have

$$\bar{\nu} = \frac{1}{\lambda} = R\,(Z-1)^2$$

or $$(Z-1)^2 = \frac{1}{R\lambda}$$

$$= \frac{3646\times10^{-10}}{4\times1\times10^{-10}}$$

$$= 911.5$$

$\therefore$ $$(Z-1) = \sqrt{911.5}$$

$$\cong 30.2$$

or $$Z = 31.2 \cong 31$$

Thus, the atomic number of the element concerned is 31. The element having atomic number Z = 31 is Gallium.

18. Option (4) is correct.

Wavelength λ of the neutron is

$$\lambda = \frac{h}{p}$$

$$= \frac{h}{\sqrt{2mE}}$$

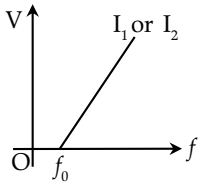
$$= \frac{6.63\times10^{-34}}{\sqrt{2\times1.67\times10^{-27}\times1.6\times10^{-19}}}$$

$$= 2.87 \times 10^{-11}\ \text{m}$$

$$\approx 10^{-8}\ \text{cm}$$

19. Option (4) is correct.

The value of stopping potential depends on the maximum kinetic energy of the photo-electron and thus it is independent of the intensity of incident light. Therefore, none of the graph shown is correct. The correct graph is shown in the figure.



20. Option (4) is correct.

Phosphorus is pentavalent impurity. Its doping will not affect the concentration of holes. So, the number of holes will be the same as in the intrinsic semiconductor. So, $n_h = 1.41 \times 10^{16}\ \text{m}^{-3}$.

21. Correct answer is [30°].

$$i = \frac{3}{4}A$$

$\Rightarrow$ $$i = 45°$$

and $$\delta_m = 2i - A$$

$$= 2 \times 45° - 60°$$

$\Rightarrow$ $$i = 30°$$

22. Correct answer is [375].

In an adiabatic process,

$$TV^{\gamma-1} = \text{constant}$$

$$\frac{T_1}{T_2} = \left(\frac{V_2}{V_1}\right)^{\gamma-1}$$

$$\frac{300}{T_2} = \left(\frac{\frac{8}{27}V}{V}\right)^{\frac{5}{3}-1}$$

$$= \left(\frac{8}{27}\right)^{\frac{2}{3}}$$

$$= \frac{4}{9}$$

$$T_2 = 300 \times \frac{9}{4}$$

$$= 675\ \text{K}$$

$\therefore$ Rise in temperature

$$= 675 - 300$$

$$= 375\ \text{K}$$

23. Correct answer is [5.00].

If C doesn't move then

$$a_A = 4a_B \quad \ldots(1)$$

$$P - T - \mu mg = m\,4a_B \quad \ldots(2)$$

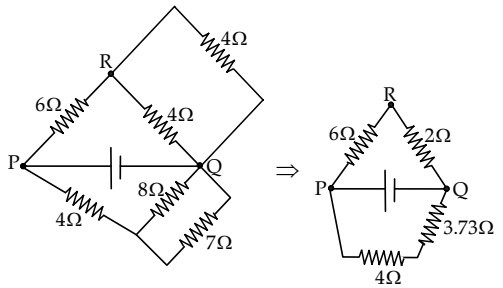
$$4T - mg = ma_B \quad \ldots(3)$$

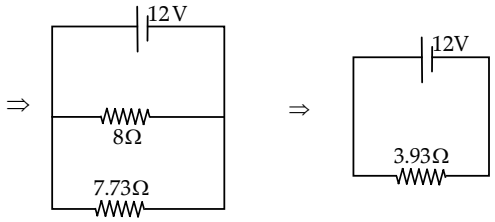
$$2T = mg \quad \ldots(4)$$

Solving above equations

$\therefore$ $$\frac{P}{mg} = 5$$

24. Correct answer is [36.00].

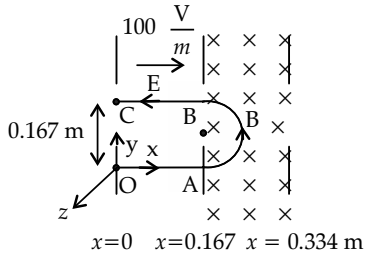




$$\text{Power dissipated} = \frac{(12)^2}{3.93} = 36 \text{ W}$$

25. Correct answer is [7.07].

The situation described in the problem is shown in the figure. As electric field is along x-axis, proton will be accelerated by the electric field and will enter the magnetic field at A (i.e., $x = 0.167$, $y = 0$) with velocity v along x-axis such that



$$\frac{1}{2}mv^2 = W = Fd = qEd$$

i.e., $$v = \left[\frac{2qEd}{m}\right]^{1/2}$$

$$= \left[\frac{2\times1.6\times10^{-19}\times100\times0.167}{1.67\times10^{-27}}\right]^{1/2}$$

$$= 4\sqrt{2}\times10^4 \frac{\text{m}}{\text{s}}$$

Now, as the proton is moving perpendicular to the magnetic field, it will describe a circular path in the magnetic field with radius r, such that

$$r = \frac{mv}{q\text{B}}$$

And as it comes back at C [$x = 0$; $y = 0.167$ m], its path in the magnetic field will be a semicircle such that

$$y = 2r = \frac{2mv}{q\text{B}}$$

i.e., $$\text{B} = \frac{2mv}{qy}$$

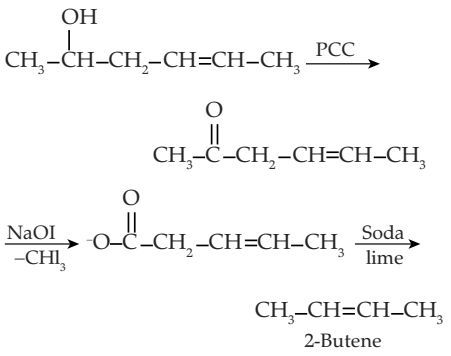
i.e., $$\text{B} = \frac{2\times1.67\times10^{-27}\times4\sqrt{2}\times10^4}{1.6\times10^{-19}\times0.167}$$

$$= \frac{1}{\sqrt{2}}\times10^{-2}$$

$$= 7.07 \text{ mT}$$

Chemistry

26. Option (3) is correct.



27. Option (1) is correct.

$$N_2(14) = (\sigma_{1s})^2(\sigma^*_{1s})^2(\sigma_{2s})^2(\sigma^*_{2s})^2$$

$$\left(\pi_{2p_x^2} = \pi_{2p_y^2}\right)(\sigma_{2p_z})^2$$

Bond order = $\frac{1}{2}$ (No. of electrons in bonding orbital —No. of electrons in anti bonding orbital)

$$\text{B.O} = \frac{1}{2}(10-4) = 3$$

$$NO^+(14) = (\sigma_{1s})^2(\sigma^*_{1s})^2(\sigma_{2s})^2(\sigma^*_{2s})^2$$

$$\left(\pi_{2p_x^2} = \pi_{2p_y^2}\right)(\sigma_{2p_z})^2$$

$$\text{B.O} = \frac{1}{2}(10-4) = 3$$

28. Option (1) is correct.

Given,

K_{sp} for bismuth sulphide (Bi_2S_3)

$$= 1.08\times10^{-7}$$

$$K_{sp}(Bi_2S_3) = [Bi^{3+}]^2\,[S^{2-}]^3$$

$$= (2S)^2\times(3S)^3$$

$$K_{sp} = 108\,S^5$$

$$S^5 = 1.08\times10^{-73}/108$$

$$S = 10^{-15}$$

29. Option (1) is correct.

The electronic configuration of these elements are:

Z = 107 [Rn] $5f^{14}\,6d^5\,7s^2$

Z = 108 [Rn] $5f^{14}\,6d^6\,7s^2$

Z = 109 [Rn] $5f^{14}\,6d^7\,7s^2$

These elements will be place in *d*-block in groups 7th, 8th and 9th respectively as they have incompletely filled *d*-orbital.

30. Option (4) is correct.

$H_2C_2O_4 \rightleftharpoons 2H^+ + C_2O_4^{2} - Ka_3$

$H_2C_2O_4 \rightleftharpoons H^+ + HC_2O_4 - Ka_1$

$HC_2O_4^- \rightleftharpoons H^+ + C_2O_4^{2} - Ka_2$

$$K_{a_3} = \frac{\left[H^+\right]^2\left[C_2O_4^{2-}\right]}{\left[H_2C_2O_4\right]}$$

$$K_{a_1} = \frac{\left[H^+\right]\left[HC_2O_4^-\right]}{\left[H_2C_2O_4\right]}, K_{a_2} = \frac{\left[H^+\right]\left[C_2O_4^{2-}\right]}{\left[HC_2O_4^-\right]}$$

$K_{a_3} = K_{a_1} \times K_{a_2}$

31. Option (2) is correct.

$\Delta H_{fusion} = 1.435 \text{ kcal mol}^{-1}$

$= 1435 \text{ cal mol}^{-1}$

$\Delta S = 5.26 \text{ cal mol}^{-1} \text{ K}^{-1}$

$$\Delta S = \frac{\Delta H_{fusion}}{T}$$

$$T = \frac{\Delta H_{fusion}}{\Delta S} = \frac{1435}{5.26}K$$

$= 272.81K \approx 0°C$

32. Option (2) is correct.

$Fe^{+3}\,[Fe\,(CN)_6]^{4-}$

Let the oxidation state of Fe in the complex be x

$x + 6\,(-1) = -4$

$x = -4 + 6$

$x = +2$

33. Option (2) is correct.

Electronic configuration of O_2^- (17 electrons) is as follows:

$\sigma 1s^2 < \sigma^* 1s^2 < \sigma 2s^2 < \sigma^* 2s^2 < \sigma 2p_z^{\,2} < \pi 2p_x^{\,2}$

$= \pi 2p_y^{\,2} < \pi^* 2p_x^{\,2} = \pi^* 2p_y^{\,1}$

It contains 1 unpaired electron thus, it is paramagnetic in nature.

34. Option (4) is correct.

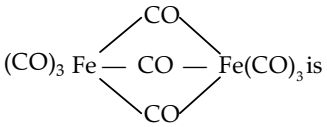
The ionisation energy of Cu is more than that of K though both have a 4*s* configuration. This is because Cu has filled the 3*d* orbital which increases the attraction between the nucleus and electrons.

35. Option (2) is correct.

$CH_3CH_2–\underset{\displaystyle CH_3}{\overset{\displaystyle |}{N}}–CH=CH_2$

Ethyl methyl vinyl amine

36. Option (1) is correct.

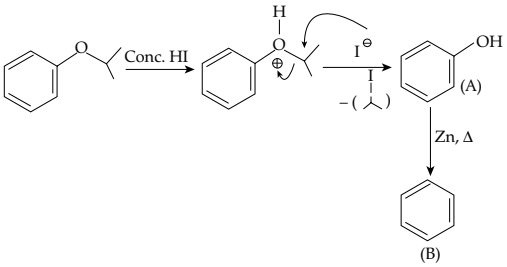


Tri-μ-carbonyl bis (tricarbonyl ion (0))

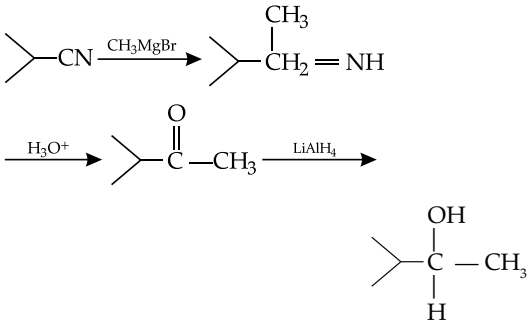
37. Option (4) is correct.

Coordination compounds having coordination number 6 of type MA_4B_2, MA_3B_3, $M(AB)_3$, $M(AA)_2B_2$ exhibit geometrical isomerism.

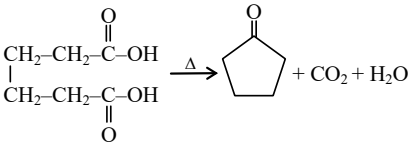
38. Option (4) is correct.



39. Option (3) is correct.



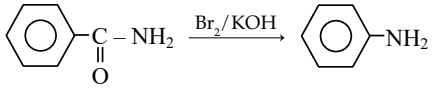
40. Option (2) is correct.



Adipic acid Cyclopentanone

Adipic acid undergo dehydration as well as decarboxylation to give cyclopentanone when it is heated.

41. Option (2) is correct.



Benzanide (A) Aniline

42. Option (2) is correct.

$(C_6H_{12}O_5)n + nH_2O \xrightarrow[\text{diastase}]{(x)} C_{12}H_{22}O_{11}$

$C_{12}H_{22}O_{11} + H_2O \xrightarrow[H_2O]{\text{Maltase (y)}} 2C_6H_{12}O_6$

43. Option (4) is correct.

From photoelectric Einstein equation:

$$E = E^\circ + K.E.$$

$$h\nu = h\nu^\circ + K.E.$$

$$K.E. = h\nu - h\nu^\circ$$

$$K.E. = h(\nu - \nu^\circ)$$

If $\nu > \nu_0$, K.E. > 0

So, if frequency of the light (ν) increases the K.E. of these electrons increases.

44. Option (1) is correct.

In the Hunsdiecker reaction →

$$CH_3CH_2COOAg + Br_2 \xrightarrow{CCl_4}$$

Silver Propionate (No. of C = 3)

$$CH_3CH_2Br + CO_2 + AgBr$$

Ethyl Bromide (No. of C = 2)

No. of carbon atoms decrease.

45. Option (3) is correct.

$$CaC_2 + 2H\text{–}OH \rightarrow Ca(OH)_2 + H - C \equiv C - H$$

(A) Acetylene

$\downarrow Hg^{2+}/H_2SO_4$

$$CH_3\text{–}CH_2OH \xleftarrow{H_2/Ni} H - \underset{\underset{O}{\|}}{C} - CH_3$$

(C) Ethanol — (B) Acetaldehyde

46. Correct answer is [10.70].

$$\mu = \frac{K_{T+10}}{K} = 1.75$$

$$\log \frac{K_{T+10}}{K} = \frac{E_a}{2.303R}\left[\frac{1}{T_1} - \frac{1}{T_2}\right]$$

$$T_1 = 25^\circ C = 298K$$

$$T_2 = 35^\circ C = 308K$$

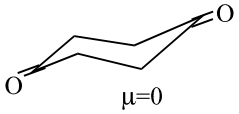
$$2.303 \times \log \frac{K_{T+10}}{K} = \frac{E_a}{1.987}\left[\frac{308 - 298}{308 \times 298}\right]$$

$$E_a = \frac{2.303 \times 308 \times 298 \times 1.987}{10} \log 1.75 \text{ cal. mol}^{-1}$$

$$= 10.2 \text{ kcal mol}^{-1}$$

$$E_a = 10.2 \text{ kcal mol}^{-1}$$

47. Correct answer is [6.0].



μ=0

Twist boat μ = 0

$$\mu_{net} = \mu_{chair}\, x_{chair} + \mu_{t.b.}\, x_{t.b.}$$

$$1.2 = 0 + \mu_{t.b.}\,(0.20)$$

$\because$ $x_{chair} = 0.80$

$\therefore$ $x_{t.b.} = 1 - 0.80 \Rightarrow 0.2$

$\Rightarrow$ $\mu_{t.b.} = 6.0$ debye

48. Correct answer is [266].

Given, $R = 1750\Omega$

$$K = 0.152 \times 10^{-3}\ \Omega^{-1}cm^{-1}$$

Conductivity = Cell constant/ Resistance

Cell constant = Conductivity × Resistance

$$= 0.152 \times 10^{-3}\ \Omega^{-1}cm^{-1} \times 1750\Omega$$

Cell constant $= 266 \times 10^{-3}\ cm^{-1}$

The cell constant depends on the distance between the electrodes and their area of cross–section

49. Correct answer is [112.70].

$$\frac{P^o - P_s}{P_s} = \frac{n}{N}, \rightarrow x_2 \text{ (mole fraction)}$$

$$\frac{P^o - P_s}{P_s} = \frac{n_2}{n_1} (\text{since } n_1 >> n_2)$$

$$= \frac{W_2}{M_2} \times \frac{M_2}{W_1}$$

$$\frac{736 - 526}{526} = \frac{2.5x}{M_2} \times \frac{18}{1x}$$

$$M_2 = 112.7 \text{ g}$$

50. Correct answer is [1.122].

$$Ph - MgBr + HC \equiv C - CH_2\text{–}CH_2\text{–}CH_2\text{–}OH$$

$\downarrow$

$$C_6H_6 + HC \equiv C\text{–}CH_2CH_2CH_2OMgBr$$

since, 84 g $HC \equiv C\text{–}CH_2\text{–}CH_2CH_2\text{–}OH$

gives → 22.4 L Benzene

$\therefore$ 1 g of Pent – 4 – yne – 1 – ol → $\frac{22.4}{84}$

$\therefore$ 4.2 g of Pent – 4 – yne – 1 – ol →

$$\frac{22.4}{84} \times 4.2 = 1.12$$

Mathematics

51. Option (2) is correct.

$$32^{32} = (2^5)^{32} = 2^{160}$$

$$= (3 - 1)^{160} = 3k + 1$$

Now, $32^{32^{32}} = (2^5)^{3k+1} = 2^{(15k+5)}$

$$= 2^{(15k+3)+2} = 4.2^{3(5k+1)}$$

$$= 4.8^{5k+1} = 4(7 + 1)^{\alpha}$$

$$= 4[7\mu + 1]$$

$\therefore$ The remainder when $32^{32^{32}}$ is divided by 7 is 4.

So, $m = 4$

Now, $\lim\limits_{x\to 0}\left(\frac{\sin x}{x}\right)^{\frac{\sin x}{x-\sin x}} = e^n$

$\Rightarrow \quad e^n = e^{\lim\limits_{x\to 0}\left[1+\frac{\sin x - x}{x}\right]\frac{\sin x}{x-\sin x}}$

$\Rightarrow \quad n = \lim\limits_{x\to 0}\left(-\frac{\sin x}{x}\right)$

$\Rightarrow \quad n = -1$

Now, the quadratic equation whose roots are $\frac{1}{m}$ and $\frac{1}{n}$ is given by $x^2 - \left(\frac{1}{m}+\frac{1}{n}\right)x + \frac{1}{mn} = 0$.

$\Rightarrow \quad x^2 - \left(\frac{1}{4}-1\right)x - \frac{1}{4} = 0$

$\Rightarrow \quad x^2 + \frac{3}{4}x - \frac{1}{4} = 0$

$\Rightarrow \quad 4x^2 + 3x - 1 = 0$

Hint:

(1) First, simplify 32^{32} using binomial theorem, then simplify $32^{32^{32}}$ and analyse further.

(2) Quadratic equation whose roots are α and β is given by $x^2 - (\alpha + \beta)x + \alpha\beta = 0$

52. Option (1) is correct.

Let $I = \int_0^1 \frac{dx}{((x-1)^2+1)^3}$

$= -\int_0^1 \frac{dx}{(x^2+1)^3}$

Let $x = \tan\theta$

$\Rightarrow \quad dx = \sec^2\theta\, d\theta$

$\int_0^{\pi/4} \frac{\sec^2\theta}{\sec^6\theta}\, d\theta$

$= \int_0^{\pi/4} \cos^4\theta\, d\theta$

$= \frac{1}{4}\int_0^{\pi/4} (1+\cos 2\theta)^2\, d\theta$

$= \frac{1}{4}\int_0^{\pi/4} (1+2\cos 2\theta+\cos^2 2\theta)\, d\theta$

$= \frac{1}{4}\int_0^{\pi/4} (1+2\cos 2\theta)\, d\theta + \frac{1}{8}\int_0^{\pi/4} (1+\cos 4\theta)\, d\theta$

$= \frac{1}{4}\left[\theta+\sin 2\theta\right]_0^{\frac{\pi}{4}} + \frac{1}{8}\left(\theta+\frac{\sin 4\theta}{4}\right)_0^{\frac{\pi}{4}}$

$= \frac{1}{4}\left[\frac{\pi}{4}+1\right] + \frac{1}{8}\left[\frac{\pi}{4}\right]$

$= \frac{3\pi+8}{32}$

53. Option (3) is correct.

Given system of linear equations:

$x + y + z = 6$

$x + 2y + 3z = 10$

$x + 2y + k_1 z = k_2$

Now, $\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 2 & k_1 \end{vmatrix}$

$= \begin{vmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 1 & 1 & k_1-2 \end{vmatrix} = k_1 - 3$

$\Delta_x = \begin{vmatrix} 6 & 1 & 1 \\ 10 & 2 & 3 \\ k_2 & 2 & k_1 \end{vmatrix}$

$= \begin{vmatrix} 0 & 1 & 0 \\ -2 & 2 & 1 \\ k_2-12 & 2 & k_1-2 \end{vmatrix}$

$= 2k_1 + k_2 - 16$

$\Delta_y = \begin{vmatrix} 1 & 6 & 1 \\ 1 & 10 & 3 \\ 1 & k_2 & k_1 \end{vmatrix}$

$= \begin{vmatrix} 1 & 6 & 1 \\ 0 & 4 & 2 \\ 0 & k_2-10 & k_1-3 \end{vmatrix}$

$= 4k_1 - 2k_2 + 8$

$\Delta_z = \begin{vmatrix} 1 & 1 & 6 \\ 1 & 2 & 10 \\ 1 & 2 & k_2 \end{vmatrix}$

$= \begin{vmatrix} 1 & 1 & 6 \\ 0 & 1 & 4 \\ 0 & 0 & k_2-10 \end{vmatrix}$

$= k_2 - 10$

For unique solution, $\Delta \neq 0 \Rightarrow k_1 \neq 3$

For infinite number of solutions,

$\Delta = 0, \Delta_x = \Delta_y = \Delta_z = 0$

$\therefore \quad k_1 = 3$ and $k_2 = 10$

For no solution, $\Delta = 0$ and at least one of Δ_x, Δ_y, Δ_z is non zero.

$\therefore \quad k_1 = 3$ and $k_2 = 10$

Hint:

(1) Find Δ, Δ_x, Δ_y and Δ_z in terms of k_1 and k_2.

(2) Unique solution, $\Delta \neq 0$.

(3) For no solution, $\Delta = 0$ and at least one of Δ_x, Δ_y, Δ_z is non zero and for infinite solutions,

$$\Delta = \Delta_x = \Delta_y = \Delta_z = 0$$

54. Option (1) is correct.

Given differential equation, $\frac{dy}{dx}(x^2y^3 + xy) = 1$

$$\Rightarrow \quad \frac{dx}{dy} = x^2y^3 + xy$$

$$\Rightarrow \quad \frac{dx}{dy} - xy = x^2y^3$$

$$\Rightarrow \quad \frac{1}{x^2}\frac{dx}{dy} - \frac{y}{x} = y^3$$

Let $-\frac{1}{x} = u$

$$\Rightarrow \quad \frac{1}{x^2}\frac{dx}{dy} = \frac{du}{dy}$$

So, $\frac{du}{dy} + uy = y^3$, which is a linear differential equation.

Now, I.F. $= e^{\int y dy} = e^{y^2/2}$

So, solution of given differential equation is

$$ue^{y^2/2} = \int y^3 e^{y^2/2} dy + c_1$$

$$\Rightarrow \quad ue^t = \int 2te^t dt + c_1 \quad \left\{\text{Putting } \frac{y^2}{2} = t\right\}$$

$$\Rightarrow \quad ue^t = 2e^t(t-1) + c_1$$

$$\Rightarrow \quad u = 2(t-1) + c_1e^{-t}$$

$$\Rightarrow \quad -\frac{1}{x} = 2\left(\frac{y^2}{2} - 1\right) + c_1e^{-y^2/2}$$

$$\Rightarrow \quad -1 = x(y^2 - 2) + xe^{-y^2/2}c_1$$

$$\Rightarrow \quad x(y^2 - 2 + c_1e^{-y^2/2}) = -1$$

$$\therefore \quad m = 2$$
$$n = -2$$
$$k = 2$$
$$l = -1$$

So, ${}^{m-n+k+l}C_m = {}^5C_2 = \frac{5.4}{2.1} = 10$

Hint: Simplify the given differential equation, substitute $-\frac{1}{x} = u$ and solve further using the concept of linear differential equation.

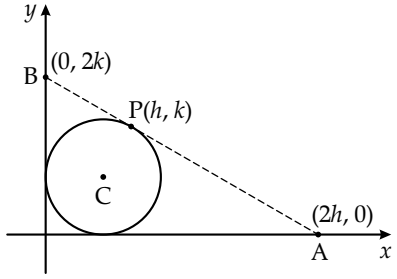
55. Option (1) is correct.

Given equation of circle is,

$$x^2 + y^2 - 2\lambda x - 2\lambda y + \lambda^2 = 0$$

$\Rightarrow$ Centre $= (\lambda, \lambda)$

and Radius $= \lambda$



Let the circumcentre be P(h, k)

As we know, circumcentre of a right angle triangle is the midpoint of hypotenuse.

$\therefore$ A $= (2h, 0)$

and B $= (0, 2k)$

Now, equation of AB is $\frac{x}{2h} + \frac{y}{2k} = 1$

$\because$ AB is tangent to circle

$\Rightarrow$ CP $= \lambda$

$$\Rightarrow \quad \lambda = \frac{\left|\frac{\lambda}{2h} + \frac{\lambda}{2k} - 1\right|}{\sqrt{\frac{1}{4h^2} + \frac{1}{4k^2}}}$$

$$\Rightarrow \quad \frac{\lambda}{2hk}\sqrt{h^2 + k^2} = |\lambda k + \lambda h - 2hk|\frac{1}{2hk}$$

$$\Rightarrow \quad \lambda^2(h^2 + k^2) = \lambda^2k^2 + \lambda^2h^2 + 4h^2k^2 + 2\lambda^2kh - 4\lambda h^2k - 4hk^2\lambda$$

$$\Rightarrow \quad 4\lambda hk(h + k) = 2hk(2hk + \lambda^2)$$

$$\Rightarrow \quad 2\lambda(h + k) = 2hk + \lambda^2$$

$$\Rightarrow \quad 2(h + k) = \frac{2hk}{\lambda} + \lambda$$

$$\Rightarrow \quad 2(x + y) = \frac{2xy}{\lambda} + \lambda$$

$$\Rightarrow \quad 2(x + y) - \lambda = \frac{2xy}{\lambda}$$

$$\Rightarrow \quad (m + n) = 4$$

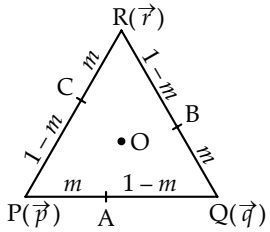
Hint:

(1) Circumcentre of right angle triangle is the mid point of hypotenuse.

(2) The perpendicular distance from the centre to the tangent is equal to the circle's radius.

56. Option (3) is correct.

Let $\vec{p}, \vec{q}, \vec{r}$ be the position vectors of P, Q, R respectively.



Given, $\frac{PA}{PQ} = \frac{QB}{QR} = \frac{RC}{RP} = m$

$\Rightarrow \quad \vec{OA} = m\vec{q} + (1-m)\vec{p}$

$\Rightarrow \quad \vec{OB} = m\vec{r} + (1-m)\vec{q}$

$\Rightarrow \quad \vec{OC} = m\vec{p} + (1-m)\vec{r}$

$\vec{PB} = \vec{OB} - \vec{OP}$

$= m\vec{r} + (1-m)\vec{q} - \vec{p}$

$\vec{QC} = m\vec{p} + (1-m)\vec{r} - \vec{q}$

$\vec{RA} = m\vec{q} + (1-m)\vec{p} - \vec{r}$

Now, area of triangle formed by $\vec{PB}$, $\vec{QC}$, $\vec{RA}$ is

$$\Delta = \frac{1}{2}|\vec{PB} \times \vec{QC} + \vec{QC} \times \vec{RA} + \vec{RA} \times \vec{PB}|$$

$$\Delta = \frac{1}{2}|m^2 - m + 1| \ (\Delta PQR)$$

So, Δ is least if $m = \frac{1}{2}$.

Hint: Assume position vector of P, Q, R, find $\vec{OA}$, $\vec{OB}$, $\vec{OC}$ and then find $\vec{PB}$, $\vec{QC}$ and $\vec{RA}$ and solve further.

57. Option (1) is correct.

$$[3 \quad x+2 \quad x+1]\begin{bmatrix} x \\ -1 \\ 1 \end{bmatrix} = 0$$

$\Rightarrow \quad 3x - (x+2) + (x+1) = 0$

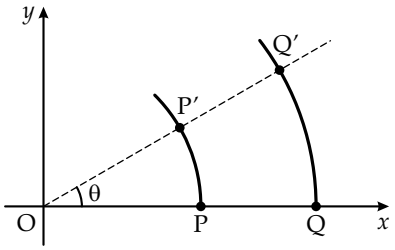
$\Rightarrow \quad 3x - 1 = 0$

$\Rightarrow \quad x = \frac{1}{3}$

58. Option (2) is correct.

Given, Points P $= (\sqrt{13}, 0)$

and $Q = \left(2\sqrt{13}, 0\right)$



$\because \quad \theta = \tan^{-1}\left(\frac{2}{3}\right)$

$\Rightarrow \quad \tan\theta = \frac{2}{3}$

$\Rightarrow \quad \sin\theta = \frac{2}{\sqrt{13}}$

and $\cos\theta = \frac{3}{\sqrt{13}}$

$\Rightarrow \quad P' = (OP\cos\theta, OP\sin\theta)$

$= (3, 2)$

Similarly, $Q' = (OQ\cos\theta, OQ\sin\theta)$

$= (6, 4)$

Now, equation of P′Q′ is

$\Rightarrow \quad y - 2 = \left(\frac{4-2}{6-3}\right)(x-3)$

$\Rightarrow \quad y - 2 = \left(\frac{2}{3}\right)(x-3)$

$\Rightarrow \quad 3y - 6 = 2x - 6$

$\Rightarrow \quad 3y - 2x = 0$

Hence, foot of perpendicular from (4, –2) to line P′Q′ is (α, β)

$\because \quad \frac{\alpha-4}{-2} = \frac{\beta+2}{3}$

$= \frac{-(3(-2)-2(4))}{(3)^2+(2)^2}$

$\frac{\alpha-4}{-2} = \frac{\beta+2}{3} = +\frac{14}{13}$

$\Rightarrow \quad \alpha = \frac{-28}{13} + 4 = \frac{24}{13}$

and $\beta = \frac{42}{13} - 2 = \frac{16}{13}$

$\Rightarrow \quad 13(\alpha + \beta) = 24 + 16 = 40$

Hint:

(1) Use P′ = (OP cos θ, OP sin θ);

where $\theta = \tan^{-1}\left(\frac{2}{3}\right)$

(2) If P (x_2, y_2) is the foot of a perpendicular drawn from Q (x_1, y_1) on the line

$$ax + by + c = 0$$

then $\frac{x_2 - x_1}{a} = \frac{y_2 - y_1}{b} = -\frac{(ax_1 + by_1 + c)}{a^2 + b^2}$

59. Option (2) is correct.

Total cases = 52

Favourable cases $x = 13 + 3 = 16 = x$

Unfavourable cases $y = 52 - 16 = 36 = y$

Odds in favour $= \frac{x}{y} = \frac{16}{36} = \frac{4}{9} = 4:9$

60. Option (3) is correct.

Let $z = x + iy$

Given, $\arg\left(\frac{z - z_1}{z - z_2}\right) = \frac{\pi}{2}$

$\Rightarrow$ z lines on the circle with z_1, z_2 as extrimities of a diameter.

$\Rightarrow$ Centre (7, 6) and radius = 3

$\therefore$ $|z - 7 - 9i| = 3\sqrt{2}$

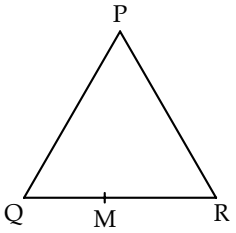
61. Option (4) is correct.

Given, P = (2, 3, 5)

Q = (– 1, 2, 2)

and R = (α, 5, β)

mid point of QR $= \left(\frac{\alpha - 1}{2}, \frac{7}{2}, \frac{\beta + 2}{2}\right)$



Direction ratios of PM

$= \frac{\alpha - 1}{2} - 2, \frac{7}{2} - 3, \frac{\beta + 2}{2} - 5$

$\because$ PM is equally inclined to the axis.

$\frac{\alpha - 5}{2} = \frac{1}{2} = \frac{\beta - 8}{2}$

$\Rightarrow$ $\alpha = 6$

and $\beta = 9$

So, $n = \alpha + \beta = 15$

Let $a = {}^nC_0 + 3{}^nC_1 + 5{}^nC_2 + \ldots.. + (2n + 1)\,C_n$

$a = \sum_{r=0}^{n}(2r+1)C_r$

$= 2\sum_{r=0}^{n} r\,C_r + \sum_{r=0}^{n} C_r$

$= 2(n\,2^{n-1}) + 2^n$

$= 2^n (n + 1)$

$\Rightarrow$ $a = 2^{15}(15 + 1) = 2^{19}$

Hint:

(1) If line $\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c}$ is equally inclined to axes, then $a = b = c$.

(2) $\sum_{r=0}^{n} r\,C_r = n2^{n-1}$ and $\sum_{r=0}^{n} C_r = 2^n$

62. Option (2) is correct.

Given, $g : R \to \left(0, \frac{\pi}{2}\right]$; $g(x) = \cot^{-1}(x^2 + x + k)$

$\because$ $g(x)$ is an onto function.

$\Rightarrow$ $0 < \cot^{-1}(x^2 + x + k) \le \frac{\pi}{2}$

$\Rightarrow$ $0 \le x^2 + x + k < \infty$

$\Rightarrow \left(x + \frac{1}{2}\right)^2 + \left(k - \frac{1}{4}\right) \ge 0$

This is possible $\forall\ x \in R$, iff $k - \frac{1}{4} = 0$

$\Rightarrow$ $k = \frac{1}{4}$

Hint: For an onto function, codomain = range.

63. Option (2) is correct.

- a, b, c are in AP

$\Rightarrow$ $2b = a + c$

$\Rightarrow$ $2ab = a^2 + ac$

$\Rightarrow$ $a^2 - 2ab = -ac$

- a, b, d in GP

$\Rightarrow$ $b^2 = ad$

$\Rightarrow$ $a^2 - 2ab + b^2 = -ac + ad$

$\Rightarrow$ $(a - b)^2 = a(d - c)$

$\Rightarrow$ $a, a - b, d - c$ are in GP

64. Option (3) is correct.

$$\frac{1}{1!\cdot(n-1)!} + \frac{1}{3!\cdot(n-3)!} + \frac{1}{5!\cdot(n-5)!} + \ldots$$

$$= \frac{1}{n!}\left[\frac{n!}{1!(n-1)!} + \frac{n!}{3!(n-3)!} + \frac{n!}{5!(n-5)!} + \ldots\right]$$

$$= \frac{1}{n!}\left[{}^nC_1 + {}^nC_3 + {}^nC_5 + \ldots\right]$$

Since,

${}^nC_0 + {}^nC_1 + {}^nC_2 + {}^nC_3 + {}^nC_4 + {}^nC_5 + \ldots + {}^nC_n = 2^n$

And ${}^nC_0 - {}^nC_1 + {}^nC_2 - {}^nC_3 + {}^nC_4 - {}^nC_5 + \ldots + (-1)^n\,{}^nC_n = 0$

Subtracting, we get

$2[{}^nC_1 + {}^nC_3 + {}^nC_5 + \ldots] = 2^n$

$\Rightarrow$ ${}^nC_1 + {}^nC_3 + {}^nC_5 + \ldots = \frac{2^n}{2} = 2^{n-1}$

$\therefore$ $\frac{n!}{1!(n-1)!} + \frac{n!}{3!(n-3)!} + \frac{n!}{5!(n-5)!} + \ldots = 2^{n-1}$

$\Rightarrow$ $\left[\frac{1}{1!(n-1)!} + \frac{1}{3!(n-3)!} + \frac{1}{5!(n-5)!} + \ldots\right] = \frac{2^{n-1}}{n!}$

65. Option (4) is correct.

Let $I = \int \frac{\cos^2 x + \sin 2x}{(2\cos x - \sin x)^2} dx$

$\Rightarrow \quad I = \int \frac{1 + 2\tan x}{(2 - \tan x)^2}$

$\Rightarrow \quad I = \int \frac{(\sec^2 x - \tan^2 x) + 2\tan x}{(2 - \tan x)^2} dx$

$\Rightarrow \quad I = \int \frac{\sec^2 x}{(2 - \tan x)^2} dx + \int \frac{(2 - \tan x)\tan x}{(2 - \tan x)^2} dx$

$\Rightarrow \quad I = \int \frac{\sec^2 x}{(2 - \tan x)^2} dx + \int \frac{\tan x\ (1 + \tan^2 x)}{(2 - \tan x)(1 + \tan^2 x)} dx$

Let $\tan x = u \Rightarrow \sec^2 x\, dx = du$

$\Rightarrow \quad I = \int \frac{du}{(2-u)^2} + \int \frac{u\,du}{(2-u)(1+u^2)}$

$\Rightarrow \quad I = \frac{1}{2-u} - \frac{2}{5}\log_e(2-u) + \frac{1}{5}\log_e(1+u^2)$

$$-\frac{1}{5}\tan^{-1} u + c$$

$\Rightarrow \quad I = \frac{1}{2 - \tan x} - \frac{2}{5}\log_e(2 - \tan x)$

$$+\frac{1}{5}\log_e \sec^2 x - \frac{1}{5}x + c$$

$\therefore \quad f(x) = 2 - \tan x$

$g(x) = \sec^2 x$

and $h(x) = x$

$m = 2$

$n = 1$

$r = 1$

So $f'(0) + g'(0) + h'(0) = 0$

Hint: Divide the numerator and denominator by $\cos^2 x$, simplify using $1 = \sec^2 x - \tan^2 x$ and substitute $\tan x = u$ and solve further.

66. Option (2) is correct.

Given, $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11\}$

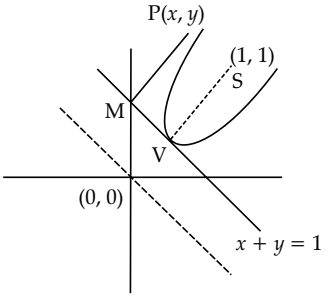
$B = \{1, 3, 4, 5, 6\}$

$\Rightarrow \quad B^C = \{2, 7, 8, 9, 10, 11\}$

$\therefore$ Number of subset of B^C which contain 10 is 2^5.

Hint: The total number of subset of finite set containing n elements is 2^n.

67. Option (1) is correct.



Distance between focus and vertex = a

$\Rightarrow \quad a = \frac{1+1-1}{\sqrt{2}} = \frac{1}{\sqrt{2}}$

Now, equation of directrix is $x + y = 0$

Since, $SP^2 = PM^2$

$\Rightarrow \quad (x-1)^2 + (y-1)^2 = \left(\frac{x+y}{\sqrt{2}}\right)^2$

$\Rightarrow \quad x^2 + y^2 - 2x - 2y + 1 + 1 = \frac{(x+y)^2}{2}$

$\Rightarrow \quad 2(x^2 + y^2 - 2x - 2y + 2) = x^2 + y^2 + 2xy$

$\Rightarrow \quad x^2 + y^2 - 2xy - 4x - 4y + 4 = 0$

68. Option (3) is correct.

Given, $f(x) = \frac{1}{\sqrt{x - 2\{x\}}} + \ln\{x\}$

Domain of $\ln\{x\}$ is R – I

$\frac{1}{\sqrt{x - 2\{x\}}}$ is exist if $x - 2\{x\} > 0$

$\Rightarrow \quad x > 2\{x\}$

$\Rightarrow \quad x \in [1, \infty]$

$\therefore$ Domain of $f(x)$ is $[1, \infty) - I^+$

Hint:

(1) $\ln x$ is defined for $x > 0$

(2) $\frac{1}{\sqrt{g(x)}}$ is defined for $g(x) > 0$

69. Option (3) is correct.

Given, hyperbolas,

H: $x^2 + 3xy + 2y^2 + 2x + 3y + 2 = 0$

and H^1: $x^2 + 3xy + 2y^2 + 2x + 3y + c = 0$

Since, H and H^1 are conjugate of each other.

$\therefore$ $H + H^1 = 0$, represents pair of straight lines.

So, $2(x^2 + 3xy + 2y^2 + 2x + 3y) + c + 2 = 0$ represents pair of straight line.

As we know, if $ax^2 + 2hxy + by^2 + 2gx + 2fy + c_1 = 0$ represents pair of straight lines, then

$abc_1 + 2fgh - af^2 - bg^2 - c_1h^2 = 0$

$\therefore$ (2) (4) (c + 2) +2 (3) (2) (3) – 2 $(3)^2$ – 4 $(2)^2$ – (c + 2) $(3)^2$ = 0

$\Rightarrow c = 0$

70. Option (3) is correct.

Total ways of selecting 5 numbers from 1, 2, 3.....18 is

$$n(S) = {}^{18}C_5$$

Arrange in the increasing order

$$x_1 < x_2 < x_3 < x_4 < x_5$$
$$\downarrow \quad \downarrow$$
$$7 \quad 11$$

choice for x_1 are {1, 2, 3, 4, 5, 6}

choice for x_3 are {8, 9, 10}

choice for x_5 are {12, 13, 14, 15, 16, 17, 18}

$$\therefore \quad n(E) = {}^6C_1 \times {}^3C_1 \times {}^7C_1$$
$$= 6 \times 3 \times 7$$

$$\therefore \quad P(E) = \frac{6\times3\times7}{{}^{18}C_5} = \frac{1}{68}$$

71. Correct answer is [80].

Given

$$\Rightarrow \sin^2 10^\circ.\sin 20^\circ.\sin 40^\circ \sin 50^\circ \sin 70^\circ = \alpha - \frac{1}{16}\sin 10^\circ$$

We have by trigonometric identity

$$\Rightarrow \sin\alpha.\sin(60^\circ - \alpha)\sin(60 + \alpha) = \frac{1}{4}\sin 30^\circ$$

$$\Rightarrow \sin 10^\circ.\frac{1}{2}(2\sin 20^\circ \sin 40^\circ)\sin 10^\circ \sin 50^\circ \sin 70^\circ = \alpha - \frac{1}{16}\sin 10^\circ$$

$$\Rightarrow \frac{1}{2}\sin 10^\circ(\cos(40^\circ - 20^\circ) - \cos(40^\circ + 20^\circ))$$
$$\sin 10^\circ \sin(60^\circ - 10^\circ)\sin(60^\circ + 10^\circ) = \alpha - \frac{1}{16}\sin 10^\circ$$

$$\Rightarrow \frac{1}{2}\sin 10^\circ(\cos 20^\circ - \cos 60^\circ).\frac{1}{4}\sin 30^\circ = \alpha - \frac{1}{6}\sin 10^\circ$$

$$\Rightarrow \frac{1}{8}\sin 10^\circ\left(\cos 20^\circ - \frac{1}{2}\right)\times\frac{1}{2} = \alpha - \frac{1}{16}\sin 10^\circ$$

$$\Rightarrow \frac{1}{16}\sin 10^\circ\left(\cos 20^\circ - \frac{1}{2}\right) = \alpha - \frac{1}{16}\sin 10^\circ$$

$$\Rightarrow \frac{1}{32}\sin 10^\circ(2\cos 20^\circ - 1) = \alpha - \frac{1}{16}\sin 10^\circ$$

$$\Rightarrow \frac{1}{32}[2\sin 10^\circ \cos 20^\circ - \sin 10^\circ] = \alpha - \frac{1}{16}\sin 10^\circ$$

$$\Rightarrow \frac{1}{32}[\sin(30^\circ) + \sin(10^\circ - 20^\circ) - \sin 10^\circ] = \alpha - \frac{1}{16}\sin 10^\circ$$

$\{\because 2\sin A\sin B = \cos(A - B) - \cos(A + B), 2\sin A\cos B = \sin(A + B) + \sin(A - B)\}$

$$\Rightarrow \frac{1}{32}\left[\frac{1}{2} - \sin 10^\circ - \sin 10^\circ\right] = \alpha - \frac{1}{16}\sin 10^\circ$$

$$\Rightarrow \frac{1}{64} - \frac{2}{32}\sin 10^\circ = \alpha - \frac{1}{16}\sin 10^\circ$$

$$\Rightarrow \frac{1}{64} - \frac{1}{16}\sin 10^\circ = \alpha - \frac{1}{16}\sin 10^\circ$$

$$\therefore \quad \alpha = \frac{1}{64}$$

Now, the value of $16 + \alpha^{-1} = 16 + \frac{1}{\alpha}$

$$= 16 + 64 = 80$$

72. Correct answer is [8.00].

$$\frac{f(\pi/4)}{1/2} = -\cos^{-1}\left(\frac{1}{\sqrt{2}}\right) - \left|f\left(\frac{\pi}{4}\right)\right|$$

$$\Rightarrow 2f\left(\frac{\pi}{4}\right) = -\left(\frac{\pi}{4} + \left|f\left(\frac{\pi}{4}\right)\right|\right)$$

$\Rightarrow f\left(\frac{\pi}{4}\right)$ is a negative quantity

$$\Rightarrow 2f\left(\frac{\pi}{4}\right) = -\frac{\pi}{4} + f\left(\frac{\pi}{4}\right)$$

$$\Rightarrow f\left(\frac{\pi}{4}\right) = -\frac{\pi}{4}$$

$$\Rightarrow \frac{f\left(\frac{\pi}{4}\right)}{\left(-\frac{\pi}{32}\right)} = \frac{-\frac{\pi}{4}}{-\frac{\pi}{32}} = 8$$

73. Correct answer is [8.00].

$$\lim_{x\to 2}\frac{2^x + 2^{3-x} - 6}{\sqrt{2^{-x}} - 2^{1-x}} = A \text{ (let)}$$

$$\Rightarrow A = \lim_{x\to 2}\frac{2^x + 8.2^{-x} - 6}{2^{-x/2} - 2^{1-x}} \quad \left(\frac{0}{0} form\right)$$

So, by L' hospital rule, we get,

$$A = \lim_{x\to 2}\frac{2^x \log 2 + 8.2^{-x}\log 2(-1) - 0}{2^{-\frac{x}{2}}\left(-\frac{1}{2}\right)\log 2 - 2.2^{-x}\log 2(-1)}$$

$$\Rightarrow A = \lim_{x\to 2}\frac{2^x - 8.2^{-x}}{-2^{-\frac{x}{2}-1} + 2^{1-x}} = \frac{2^2 - 8/4}{-2^{-2} + 2^{-1}}$$

$$= \frac{4-2}{-\frac{1}{4}+\frac{1}{2}}$$

$$= \frac{2}{1/4} = 8$$

74. Correct answer is [11].

Given, $|z_1| = 6$

$\Rightarrow$ z_1 lies on circle $x^2 + y^2 = 6^2$

and $|z_2 - 4| + |z_2 + 4| = 10$

$\Rightarrow$ z_2 lies on ellipse whose length of major axis is 10 and having foci at $(\pm 4, 0)$

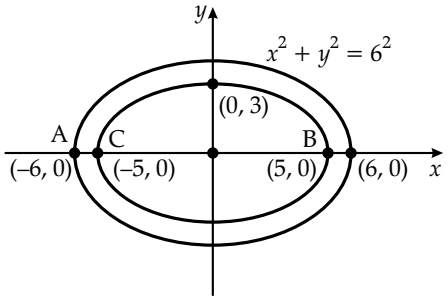
$\Rightarrow$ z_2 lies on ellipse $\frac{x^2}{5^2}+\frac{y^2}{3^2} = 1$

So, $|z_1 - z_2|_{\max} = AB = 11$

Hint:

(1) $|z - z_1| = r$, represent circle of radius r having centre at z_1.

(2) $|z - z_1| + |z - z_2| = 2a$, represents equation of ellipse whose length of major axis is $2a$ and having foci at z_1 and z_2.



75. Correct answer is [2.00].

$$(e-1)e^{xy} + x^2 = e^{x^2+y^2}$$

$$(e-1)e^{xy}\{xy' + y\} + 2x = e^{x^2+y^2}(2x + 2yy')$$

$\Rightarrow$ $$y' = \frac{(e-1)y.e^{xy} + 2x - 2x.e^{x^2+y^2}}{2ye^{x^2+y^2} - (e-1)e^{xy}.x}$$

$\Rightarrow$ $$\left.\frac{dy}{dx}\right|_{(1,0)} = \frac{0+2-2\cdot e}{0-(e-1)}$$

$$= \frac{2(1-e)}{1-e} = 2$$

❐❐

MOCK TEST PAPER 3

General Instructions: *Same as Mock Test Paper 1.*

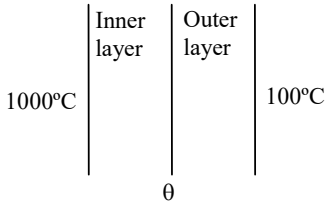
Physics

Section A

Q. 1. If 50 Vernier divisions are equal to 49 main scale divisions of a travelling microscope and the smallest reading of the main scale is 0.5 mm, the Vernier constant of travelling microscope is:
(1) 0.1 mm **(2)** 0.1 cm **(3)** 0.01 cm **(4)** 0.01 mm

Q. 2. A vessel is filled with a gas at a pressure of 76 cm of mercury at a certain temperature. The mass of the gas is increased by 50% by introducing more gas into the vessel at the same temperature. The resultant pressure, in cm of Hg, is:
(1) 76 **(2)** 152 **(3)** 114 **(4)** 1117

Q. 3. The temperature drop through a two-layer furnace wall is 900°C. Each layer is of equal area of cross-section. Which of the following actions will result in a decrease in the temperature θ of the interface?

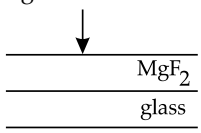


(1) By increasing the thermal conductivity of the outer layer
(2) By increasing the thermal conductivity of the inner layer
(3) By increasing thickness of outer layer
(4) By decreasing thickness of inner layer

Q. 4. In a resonance pipe, the first and second resonances are obtained at lengths 22.7 cm and 70.2 cm, respectively. Which of the following is an appropriate estimate of the end correction of the pipe?
(1) 1.05 cm **(2)** 115.5 mm
(3) 92.5 mm **(4)** 2.135 cm

Q. 5. A long solenoid has 1000 turns. When a current of 4 A flows through it, the magnetic flux linked with each turn of the solenoid is 4×10^{-3} Wb. The self-inductance of the solenoid is:
(1) 4 H **(2)** 3 H **(3)** 2 H **(4)** 1 H

Q. 6. White light is incident normally on a glass surface (n = 1.52) coated with a film of MgF_2 (n = 1.38). At what minimum thickness of the film will yellow light of wavelength 550 nm (in air) be missing in the reflected light?



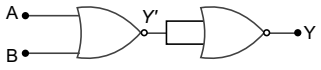
(1) 99.6 nm **(2)** 49.8 nm
(3) 19.6 nm **(4)** 10.6 nm

Q. 7. A uniform magnetic field of 2×10^{-3} T acts along a positive Y-direction. A rectangular loop of sides 20 cm and 10 cm with a current of 5A is in the Y-Z plane. The current is in an anticlockwise sense with reference to the negative X axis. Magnitude and direction of the torque is:
(1) 2×10^{-4} N – m along positive Y-direction
(2) 2×10^{-4} N – m along positive Z-direction
(3) 2×10^{-4} N – m along positive X-direction
(4) 2×10^{-4} N – m along negative Z-direction

Q. 8. A mass m is revolving in a vertical circle at the end of a string of length 20 cm. By how much does the tension of the string at the lowest point exceed the tension at the top most point?
(1) 2 mg **(2)** 4 mg **(3)** 6 mg **(4)** 8 mg

Q. 9. In Young's experiment, the ratio of maximum and minimum intensities in the fringe system is 9:1. The ratio of amplitudes of coherent sources is:
(1) 9:1 **(2)** 3:1 **(3)** 2:1 **(4)** 1:1

Q. 10. In the following circuit, the output Y for all possible inputs A and B is expressed by the truth table:



(1)

A	B	Y
0	1	1
0	1	1
1	0	1
1	1	0

(2)

A	B	Y
0	0	1
0	1	0
1	0	0
1	1	0

(3)

A	B	Y
0	1	1
1	0	1
0	0	0
1	1	1

(4)

A	B	Y
0	0	1
0	1	0
1	0	0
1	1	1

Q. 11. Moment of inertia of a uniform circular disc about a diameter is I. Its moment of inertia about an axis perpendicular to its plane and passing through a point on its rim will be:
(1) 5I **(2)** 3I **(3)** 6I **(4)** 4I

Q. 12. Given below are two statements:

Statement I: The ratio of the inertial mass to gravitational mass is equal to 1.

Statement II: The Equivalence Principle states that inertial mass and gravitational mass may differ significantly in certain situations.

Choose the correct answer from the options given below:

(1) Statement I is true, Statement II is true and Statement II is the correct explanation of Statement I.

(2) Statement I is true, Statement II is true, but Statement II is not the correct explanation of Statement I.

(3) Statement I is true, Statement II is false.

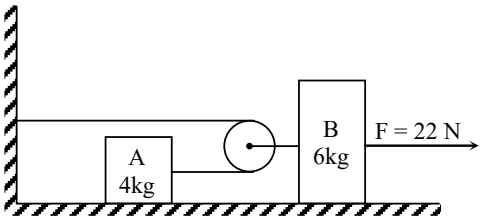
(4) Statement I is false, Statement II is true.

Q. 13. Stopping potentials of 24, 100, 110 and 115 kV are measured for photoelectrons emitted from a certain element when it is irradiated with monochromatic X-rays. If the element is used as a target in an X-ray tube, the energy of Kα line is:
(1) 54 KeV **(2)** 76 KeV
(3) 88 KeV **(4)** 32 KeV

Q. 14. A heavy nucleus with mass number 200 gets disintegrated into two small fragments with mass number 80 and 120. If binding energy per nucleon for the parent atom is 6.5 MeV and for daughter nuclei is 7 MeV and 8 MeV, respectively, then the energy released in the decay will be:
(1) 200 MeV **(2)** –220 MeV
(3) 220 MeV **(4)** 180 MeV

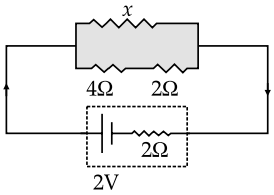
Q. 15. A particle of mass 'm' is executing oscillations about the origin on the X-axis. Its potential energy is $U(x) = k|x|^3$ where k is a positive constant. If the amplitude of oscillation is 'a', then its time period T is:
(1) Proportional to $\frac{1}{\sqrt{a}}$ **(2)** Independent of a
(3) Proportional to $\sqrt{a}$ **(4)** Proportional to $a^{3/2}$

Q. 16. Two blocks are connected by a massless string through an ideal pulley, as shown. A force of 22 N is applied on block B when the blocks are at rest. The acceleration of the centre of mass of block A and block B, 2 s after the application of force is (masses of A and B are 4 kg and 6 kg, respectively and surfaces are smooth):



(1) 1.4 m/s^2 **(2)** 1 m/s^2
(3) 2 m/s^2 **(4)** None of these

Q. 17. Observe the following circuit.



In order to ensure that the power dissipated in external circuit is maximum, the value of x should be (in ohm) _____ .
(1) 4Ω **(2)** 3Ω **(3)** 2Ω **(4)** 1Ω

Q. 18. In a series LCR circuit, voltage drop across resistance is 8 V; across inductor is 6 V and across capacitor is 12 V. Then:

(1) Voltage of the source will be leading current in the circuit.

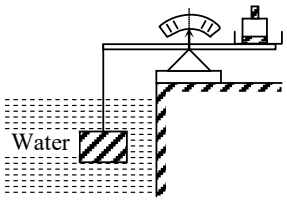
(2) Voltage drop across each element will be less than the applied voltage.

(3) Power factor of the circuit will be $\frac{4}{3}$.

(4) None of these

Q. 19. If an electron revolves around a proton, then its time period (R = radius of orbit):
(1) T ∝ R^2 **(2)** T ∝ R$^{3/2}$
(3) T ∝ R^3 **(4)** T ∝ R

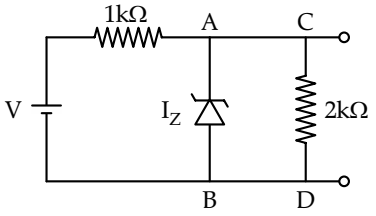
Q. 20.



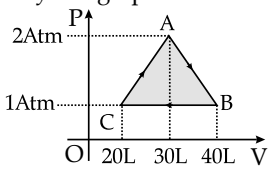
The volume of a brick is 2.197 litres. The submerged brick is balanced by a 2.54 kg mass on the beam scale. The weight of the brick is: (g = 9.8 m/s^2)
(1) 46 N **(2)** 50 N **(3)** 56 N **(4)** 72 N

Section B

Q. 21. In the given circuit, the breakdown voltage of Zener diode is 3.0 V. The value of Iz will be .....mA.

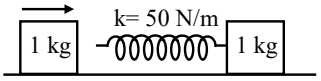


Q. 22. Consider a PV–cyclic process ABCA as per described by the graph below.



Heat of 1205.0 J is provided for the process to take place. What is the amount of heat rejected (in joule) in one cycle?

Q. 23. Each of the blocks shown in the figure has mass 1 kg. The rear block moves with a speed of 2 m/s towards the front block kept at rest. The spring attached to the front block is light and has a spring constant 50 N/m. The maximum compression of the spring is given by $\frac{X}{10}$ m, then the value of X is ..... m.



Q. 24. In a certain region of space, electric potential V is given by $V = ax^2 + ay^2 + 2az^2$ (where 'a' is a constant of proper dimensions). Work done by electric field in moving a 2μC charge from origin to (0, 0, 0.1 m) is (-5×10^{-8}) J. The approximate value of 'a' in V/m^2 is ..... .

Q. 25. A wire of length 2m is clamped horizontally between two fixed supports. A mass m = 5 kg is hung from the middle of the wire. The vertical depression in wire in equilibrium is ..... cm. (Young modulus of wire = 2.4×10^9 N/m^2, cross-sectional area = 1 cm^2)

Chemistry

Section A

Q. 26. For a molecule Br_2, total distance between two nuclei is 3.2 Å. What will be the covalent radius of Br atom?

(1) 1.6 Å **(2)** 6.4 Å **(3)** 2.4 Å **(4)** 4.9 Å

Q. 27. Match the items under list (1) with items under list (2) and select the correct answers from the sets (1), (2), (3) and (4):

List (1) Molecule	List (2) Shape
(a) PCl_5	**(i)** V-shaped
(b) F_2O	**(ii)** Triangular planar
(c) BCl_3	**(iii)** Trigonal bipyramidal
(d) NH_3	**(iv)** Trigonal pyramidal
	(v) Tetrahedral

(1) a-i, b-v, c-iv, d-iii **(2)** a-ii, b -iii, c-i , d-ii
(3) a-iv, b-iii, c-ii, d-v **(4)** a-iii, b-i, c-ii, d-iv

Q. 28. During the qualitative analysis of a salt with cation y^{2+}, the addition of a reagent (X) to an alkaline solution of the salt gives a bright red precipitate. The reagent (X) and the cation (y^{2+}) present respectively are:

(1) Dimethylglyoxime and Ni^{2+}
(2) Dimethylglyoxime and Co^{2+}
(3) Nessler's reagent and Hg^{2+}
(4) Nessler's reagent and Ni^{2+}

Q. 29. Calculate the work done when 2 moles of hydrogen expand isothermally and reversibly at 27°C from 15 to 50 litres.

(1) 14.45 k cal **(2)** 1445 J
(3) –1445 cal **(4)** 14.45 kJ

Q. 30. The k_{sp} for bismuth sulphide (Bi_2S_3) is 1.08×10^{-73}. The solubility of Bi_2S_3 in mol/L at 198 K is

(1) 2.7×10^{-12} **(2)** 1.0×10^{-15}
(3) 4.2×10^{-8} **(4)** 3.2×10^{-10}

Q. 31. Calculate the pH of 0.1 M CH_3COOH

(Given $K_{a_{CH_3COOH}} = 2 \times 10^{-5}$)

(1) 2.5 **(2)** 2.2
(3) 2.85 **(4)** 3.15

Q. 32. IUPAC name of:

$$H_3C - \underset{\displaystyle C_2H_5}{\underset{|}{CH}} - CH_2 - \underset{\displaystyle OH}{\underset{|}{CH}} - CH_2Cl$$ is:

(1) 1-chloro-4-methylhexan-2-ol
(2) 1-chloro-4-methylhexan-2-al
(3) 1-chloro-4-ethylpentan-2-ol
(4) 1-chloro-2-hydroxy-4-methylhexane

Q. 33. Given below are two statements:

Statement I: The number of geometrical isomers in asymmetrical molecule are 2^n.

Statement II: To show geometrical isomerism, molecule must be plannar and have restricted rotation.

(1) Both Statement I and Statement II are true.
(2) Both Statement I and Statement II are false.
(3) Statement I is true but Statement II is false.
(4) Statement I is false but Statement II is true.

Q. 34. Given below are two statements.

Statement I: The presence of weaker π–bonds make alkenes less stable than alkanes.

Statement II: The strength of the double bond is greater than that of the carbon-carbon single bond.

In the light of the above statements, choose the *correct* answer from the options given below.

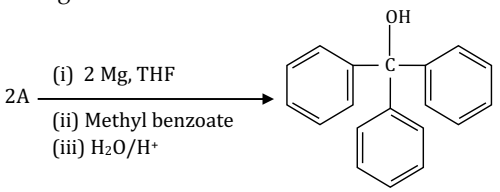
(1) Both Statement I and Statement II are correct.

(2) Both Statement I and Statement II are incorrect.

(3) Statement I is correct but Statement II is incorrect.

(4) Statement I is incorrect but Statement II is correct.

Q. 35. In the given reaction



(1) Benzyl bromide **(2)** bromobenzene

(3) cyclohexyl bromide **(4)** methyl bromide

Q. 36. Cerium (IV) has a noble gas configuration. Which of the following is correct statement about it?

(1) It will not prefer to undergo redox reactions.

(2) It will prefer to gain electron and act as an oxidising agent.

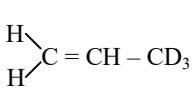
(3) It will prefer to give away an electron and behave as reducing agent.

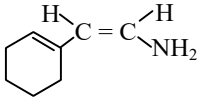
(4) It acts as both, oxidising and reducing agent.

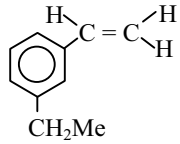
Q. 37. The depression in freezing point observed for a formic acid solution of concentration 0.5 mol L^{-1} is 0.0405°C. The density of formic acid is 1.05 g mL^{-1}. The Van't Hoff factor of the formic acid solution is nearly: (Given for water, k_f = 1.86 K kg mol^{-1})

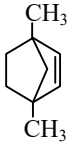
(1) 0.8 **(2)** 1.1 **(3)** 1.9 **(4)** 2.4

Q. 38. Hyperconjugation is possible in:

(i) 

(ii) 

(iii) 

(iv) 

(1) i and ii **(2)** i, ii and iii

(3) only ii **(4)** All of the above

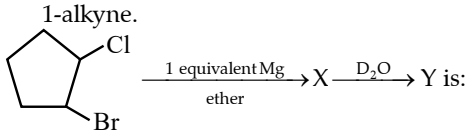
Q. 39. When 1-alkyne is treated with Na + Liq. NH_3 and product is reacted with methyl chloride, the end product of the reaction will be:

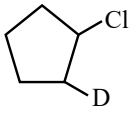
(1) lower alkyne having two carbon less than 1-alkyne.

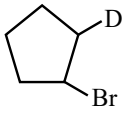
(2) lower alkyne having one carbon less than 1-alkyne.

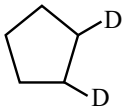
(3) higher alkyne having one carbon more than 1-alkyne.

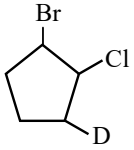
(4) higher alkyne having two carbon more than 1-alkyne.

Q. 40. 

(1) 

(2) 

(3) 

(4) 

Q. 41. In 'nitration mixture' concentrated sulphuric acid is used:

(1) as sulphonating agent.

(2) as dehydrating agent.

(3) for the formation of nitronium ions.

(4) as a solvent.

Q. 42. Methanol and ethanol are distinguished by:

(1) Treating with Victor Meyer test

(2) Treating with Lucas reagent

(3) Heating with iodine and alkali

(4) Treating with CrO_3 in dil. H_2SO_4

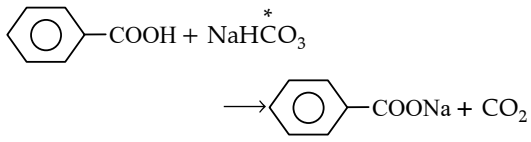
Q. 43. In which of the following crossed aldol condensations, only one kind of cross aldol is formed?

(1) CH_3CHO and CH_3CH_2CHO

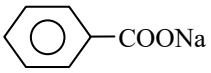
(2) CH_3CHO and $(CH_3)_2CO$

(3) $(CH_3)_2CO$ and $(C_2H_5)_2CO$

(4) C_6H_5CHO and CH_3CHO

Q. 44. 

C* is with in the product—

(1) CO_2 **(2)** 

(3) Both (1) and (2) **(4)** None of these

Q. 45. Name the products in the acid- base reaction:

(A) $CH_3CH_2NH_2$ + HI

(B) $(CH_3)_3N$ + HBr

(1) **(A)** Trimethyl ammonium iodide
(B) Trimethyl ammonium bromide

(2) **(A)** Ethyl ammonium iodide
(B) Methyl ammonium bromide

(3) **(A)** Ethyl ammonium iodide
(B) Trimethyl ammonium bromide

(4) All of the above

Section B

Q. 46. Two substances, A ($t_{1/2}$ = 5 min) and B ($t_{1/2}$ = 15 min) are taken in such a way that initially [A] = 4[B]. The time after which both the concentration will be equal is ..... (assuming reactions are of first order).

Q. 47. The standard free energy change for the reaction:

$H_2(g) + 2AgCl(s) \rightarrow 2Ag_{(s)} + 2H^+_{(aq)} + 2Cl^-_{(aq)}$

is –10.26 kcal mol^{-1}at 25°C. A cell using above reaction is operated at 25°C under P_{H_2} = 1 atm, $[H^+]$ and $[Cl^-]$ = 0.1. The e.m.f. of cell is ..... V.

Q. 48. In the given sequence, identify the no. of *c*-atom present in the final product (D).

$$CH_3CH_2OH \xrightarrow{[O]} A \xrightarrow[NaOH]{I_2} B + C$$

$$\downarrow \text{Silver powder}$$

$$D$$

Q. 49. For the reaction

$H_2F_2(g) \rightarrow H_2(g) + F_2(g)$,

$\Delta U = -59.6$ kJ mol^{-1} at 27 °C.

The enthalpy change for the above reaction is (–) _____ kJ mol^{-1} [nearest integer]

Given: R = 8.314 J K^{-1} mol^{-1}.

Q. 50. **(a)** The number of unpaired e^- in $[FeF_6]^{3-}$ is *x*.

(b) The number of geometrical isomers in $[Pt\ Cl\ Br\ IF]^{2-}$ is *y*.

(c) The oxidation state of central atom in $Ni(CO)_4$ is Z.

Find the sum value of *x*, *y* and *z*.

Mathematics

Section A

Q. 51. The minimum number of elements that must be added to the relation R = {(*a*, *b*), (*b*, *c*)} on the set {*a*, *b*, *c*}, so that it becomes symmetric and transitive is:

(1) 3 **(2)** 7 **(3)** 5 **(4)** 4

Q. 52. If D, E, F are the mid points of the sides BC, CA and AB respectively of a triangle ABC and 'O' is any point, then, $|\overrightarrow{AD} + \overrightarrow{BE} + \overrightarrow{CF}|$, is:

(1) 1 **(2)** 0 **(3)** 2 **(4)** 4

Q. 53. If $I_m = \int \frac{x^m}{y} dx$, $y^2 = 2x^2 + 4x + 3$ and constants *a*, *b*, *c* such that $aI_3 + bI_2 + cI_1 = x^2y$, then ${}^{a+c}P_{b+1}$ is equal to:

(1) 11! **(2)** 10! **(3)** 12! **(4)** 14!

Q. 54. $\int_0^\infty \left[\frac{2}{e^x}\right] dx$ is equal to, where [.] denotes greatest integer function.

(1) e^2 **(2)** $\log_e 2$ **(3)** 0 **(4)** $\frac{1}{e}$

Q. 55. Area bounded by the curves $x = |y^2 - 1|$ and $y = x - 5$ is equal to:

(1) $\frac{109}{6}$ sq. units **(2)** $\frac{121}{6}$ sq. units

(3) $\frac{113}{6}$ sq. units **(4)** $\frac{117}{6}$ sq. units

Q. 56. $\int \sqrt{1+x^{2n}} \left(\frac{\log_e(1+x^{2n}) - 2n\log_e x}{x^{3n+1}} \right) dx, n \in a$

equals to:

(1) $-\frac{2}{3n}\frac{\sqrt{(1+x^{2n})^3}}{x^{3n}}\left[\frac{1}{2}\log_e(1+x^{2n}) - \frac{1}{2}\log_e x^n + \frac{1}{3}\right] + c$

(2) $-\frac{2}{3n}\frac{\sqrt{(1+x^{2n})^3}}{x^{3n}}\left[\log_e(1+x^{2n}) - \log_e x^{2n} - \frac{1}{3}\right] + c$

(3) $-\frac{2}{3n}\frac{\sqrt{(1+x^{2n})^3}}{x^{3n}}\left[\frac{1}{2}\log_e(1+x^{2n}) - \frac{1}{2}\log_e x^{2n} - \frac{1}{3}\right] + c$

(4) $-\frac{2}{3n}\frac{\sqrt{(1+x^{2n})^3}}{x^{3n}}\left[\frac{1}{2}\log_e(1+x^{2n}) + \frac{1}{2}\log_e x^{2n} + \frac{1}{3}\right] + c$

Q. 57. If $y = a\log|x| + bx^2 + x$ has its extremum values at $x = -1$ and $x = 2$, then:

(1) $a = 2, b = -1$ **(2)** $a = 2, b = \frac{-1}{2}$

(3) $a = -2, b = \frac{1}{2}$ **(4)** a = –2, $b = \frac{-1}{2}$

Q. 58. If $x = a[\cos\theta + \log\tan\frac{\theta}{2}]$, $y = a\sin\theta$ then $\frac{dy}{dx}$ =

(1) $\cos\theta$ **(2)** $\sin\theta$ **(3)** $\tan\theta$ **(4)** $\text{cosec}\,\theta$

Q. 59. If the range of the function $y = \log_{\sqrt{2}}(2 - \log_2(16\sin^2 x + 1))$ is (*q*, P] then the value of P is:

(1) $\sqrt{2}$ **(2)** 4 **(3)** 2 **(4)** ∞

Q. 60. The smallest value of k^2 for which $\lim_{x\to 1} \sec^{-1}\left(\frac{k^2}{\log_e x} - \frac{k^2}{x-1}\right)$ exists:

(1) 1 **(2)** 2 **(3)** 4 **(4)** 3

Q. 61. A circle touching the x-axis at (3, 0) and making an intercept of length 8 on the y-axis passes through the point:

(1) (3, 10) **(2)** (3, 5)

(3) (2, 3) **(4)** (1, 5)

Q. 62. **Statement 1:** If $\sum_{i=1}^{8}(x_i-7) = 8$ and $\sum_{i=1}^{8}(x_i-7)^2 = 40$, then standard deviation of $x_1, x_2, x_3, ..., x_8$ is 2.

Statement 2: Standard deviation is independent of change of origin.

(1) Statement 1 is true, Statement 2 is true and Statement 2 is correct explanation for Statement-1.

(2) Statement 1 is true, Statement 2 is true and Statement 2 is not correct explanation for Statement 1.

(3) Statement 1 is true, Statement 2 is false.

(4) Statement 1 is false, Statement 2 is true.

Q. 63. Let a perpendicular AB be drawn from A (1, 6, 3) to the line $\frac{x}{1} = \frac{y-1}{1} = \frac{z-2}{3}$, where B is foot of perpendicular, then:

(1) B is (2, 3, 5)

(2) Image of point A *w.r.t.* given line is (1, 0, 4)

(3) Projection of the line segment AB on a line having direction cosines $\frac{1}{\sqrt{2}}, 0, -\frac{1}{\sqrt{2}}$ is $\sqrt{2}$.

(4) AB = $\sqrt{12}$

Q. 64. Let the foci of the ellipse $\frac{x^2}{16}+\frac{y^2}{7}=1$ and the hyperbola $\frac{x^2}{144}-\frac{y^2}{\alpha}=\frac{1}{25}$ coincide. Then the length of the latus rectum of the hyperbola is:

(1) $\frac{32}{9}$ **(2)** $\frac{18}{5}$ **(3)** $\frac{27}{4}$ **(4)** $\frac{27}{10}$

Q. 65. For the roots of the equation $a - bx - x^2 = 0$; ($a > 0, b > 0$), which statement is true?

(1) both roots are positive

(2) both roots are negative

(3) roots have opposite sign, negative root has greater magnitude

(4) roots have opposite sign, positive root has greater magnitude

Q. 66. Consider the system of linear equations:

$$-x + y + 2z = 0$$
$$3x - ay + 5z = 1$$
$$2x - 2y - az = 7$$

Let P be the set of all $a \in R$ for which the system is inconsistent and Q be the set of all $a \in R$ for which the system has infinitely many solutions. If ΣP and ΣQ denotes the sum of all elements of set P and set Q respectively, then $\Sigma P + \Sigma Q$ equals to:

(1) 2 **(2)** 7 **(3)** 10 **(4)** 14

Q. 67. If the ratio of the sum of n terms of two AP's is $2n : (n+1)$, then ratio of their 8^{th} terms is

(1) 15 : 8 **(2)** 8 : 13 **(3)** 11 : 6 **(4)** 5 : 17

Q. 68. If the 4^{th} term in the expansion of $\left(ax+\frac{1}{x}\right)^n$ is $\frac{5}{2}$, then the values of a and n respectively are:

(1) 2, 6 **(2)** $\frac{1}{2}$, 6

(3) $\frac{1}{2}$, 5 **(4)** 2, 5

Q. 69. The number of positive integers satisfying the inequality ${}^{n+1}C_{n-2} - {}^{n+1}C_{n-1} \le 100$ is:

(1) Nine **(2)** Eight **(3)** Five **(4)** Ten

Q. 70. If the orthocentre of the triangle formed by (1, 3) (4, –5) and (a, b) is (2, 4), Then the value of $33b + 22a$ is:

(1) 0 **(2)** $\frac{1}{11}$ **(3)** 1 **(4)** $\frac{3}{11}$

Section B

Q. 71. A line with direction ratios <0, 2, –1> meets the line $\frac{x+3}{5} = \frac{y-1}{2} = \frac{z+4}{3}$ and $\frac{x-1}{1} = \frac{y+2}{3} = \frac{z-2}{-2}$ at P and Q respectively. If area of triangle OPQ, where O is origin, is equal to $\frac{1}{n}\sqrt{m}$, then ($m - n$) equals to ... .

Q. 72. In a workshop, there are five machines and the probability of any one of them to be out of service on a day is $\frac{1}{4}$. If the probability that at most two machines will be out of service on the same day is $\left(\frac{3}{4}\right)^3 k$, then gk is equal to.....

Q. 73. If $m = \int_{-1}^{10} \text{sgn}(x-[x])dx$

and $n = \int_0^{26} \frac{e^{\cos\pi\{x\}}}{e^{\cos\pi\{x\}}+e^{-\cos\pi\{x\}}}dx$, then ${}^nc_m$ equals to; where [.] and {.} denote greatest integer function & fraction part function respectively.

Q. 74. If $D=\begin{bmatrix} 0 & a\alpha^2 & a\beta^2 \\ b\alpha+c & 0 & a\gamma^2 \\ b\beta+c & (b\gamma+c) & 0 \end{bmatrix}$ is a skew symmetric matrix (where α, β, γ are distinct) and the value of $\begin{vmatrix} (a+1)^2 & (1-a) & (2-c) \\ (3+c) & (b+2)^2 & (b+1)^2 \\ (3-b)^2 & b^2 & (c+3) \end{vmatrix}$ is λ, then the value of $|10\lambda|$ is.....

Q. 75. Let $|z| = |z-3| = |z-4i|$, then the value $|2z|$ is.....

Answers

Physics

Q. No.	Answer	Topic's name	Q. No.	Answer	Topic's name
1	(3)	Experimental Physics	14	(3)	Nuclear Fission
2	(3)	Kinetic Theory of Gases	15	(1)	Unit and Measurement
3	(1)	Heat Transfer	16	(1)	Laws of Motion
4	(1)	Sound Wave	17	(2)	Electric Circuit, Electrical Power
5	(4)	Self Inductance	18	(4)	Series LCR Circuit
6	(1)	Wave Optics	19	(2)	Atoms
7	(4)	Torque on a Current-carrying loop placed in a Magnetic field	20	(1)	Mechanical properties of Fluids
8	(3)	Motion in a Vertical Circle	21	[5.5]	Electronic Devices
9	(3)	Young's Double Slit Experiment	22	[205.00]	Thermodynamics
10	(3)	Logic Gates	23	[2.00]	Conservation of Mechanical energy
11	(3)	Moment of Inertia	24	[1.20]	Electrostatic Potential
12	(3)	Gravitation	25	[4.68]	Mechanical properties of Solids
13	(2)	Stopping Potential, X - Rays			

Chemistry

Q. No.	Answer	Topic's name	Q. No.	Answer	Topic's name
26	(1)	Periodic Table	39	(3)	Hydrocarbon
27	(4)	Chemical Bonding	40	(1)	Halogen Dervitative
28	(1)	Salt Analysis	41	(3)	P-block
29	(3)	Thermodynamics	42	(3)	Alcohol, Ether and Phenol
30	(2)	Ionic Equilibrium	43	(4)	Aldehyde and Ketone
31	(3)	Ionic Equilibrium	44	(1)	Carboxylic Acid
32	(1)	IUPAC	45	(3)	Nitrogen Compound
33	(1)	Isomerism	46	[15.00]	Chemical Kinetics
34	(1)	Hydrocarbon	47	[0.34]	Electrochemistry
35	(2)	Ketones	48	[2]	Alcohol
36	(2)	Lanthanoids	49	[57]	Thermodynamics
37	(3)	Solutions	50	[8]	Coordinaion Chemistry
38	(2)	General Organic Chemistry			

Mathematics

Q. No.	Answer	Topic's name	Q. No.	Answer	Topic's name
51	(2)	Set Theory, Relation	64	(4)	Hyperbola
52	(2)	Vector	65	(3)	Quadratic Equations
53	(3)	Indefinite Integration	66	(2)	Determinants
54	(2)	Definite Integration	67	(1)	Arithmetic Progression
55	(1)	Area under the curve	68	(2)	Binomial Theorem
56	(3)	Indefinite integration	69	(1)	Permutations and Combinations
57	(2)	Application of Derivatives – Maxima & Minima	70	(3)	Point and Straight Line
58	(3)	Differentiation	71	[19]	Three Dimensional Geometry
59	(3)	Functions	72	[17]	Probability
60	(2)	Limits	73	[78]	Definite Integral
61	(1)	Circles	74	[600]	Determinants
62	(1)	Statistics	75	[5.00]	Complex Numbers
63	(3)	Three Dimensional Geometry			

ANSWERS WITH EXPLANATION

Physics

1. Option (3) is correct.

$$\text{The least count} = \frac{1}{50}$$

So, $$\text{Vernier constant} = \frac{1}{50} \times 0.5 \text{ mm}$$

$$= 0.01 \text{ mm}$$

2. Option (3) is correct.

$$P \propto m$$

Since m is increased by a factor of $\frac{4}{3}$,

P will increase by a factor of $\frac{4}{3}$.

$$\therefore \quad \text{New pressure} = \frac{3}{2} \times 76 \text{ cm of Hg}$$

$$= 114 \text{ cm of Hg}$$

3. Option (1) is correct.

$$H = \text{rate of heat flow}$$

$$= \frac{900}{\frac{l_i}{K_i A} + \frac{l_0}{K_0 A}}$$

Now $$1000 - \theta = \frac{H l_i}{K_i A}$$

or $$\theta = 1000 - \left[\frac{900}{\frac{l_i}{K_i A} + \frac{l_0}{K_0 A}}\right] \frac{l_i}{K_i A}$$

$$= 1000 - \frac{900}{1 + \frac{l_0}{K_0}\frac{K_i}{l_i}}$$

Now, we can see that θ can be decreased by increasing thermal conductivity of outer layer (K_0) and thickness of inner layer (l_i).

4. Option (1) is correct.

$$e = \frac{l_2 - 3l_1}{2}$$

$$e = \frac{70.2 - 3 \times 22.7}{2} = \frac{70.2 - 68.1}{2}$$

$$= \frac{2.1}{2} = 1.05 \text{ cm}$$

5. Option (4) is correct.

Self-inductance of coil:

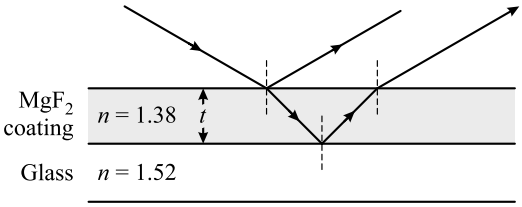
$$L = \frac{N\phi}{I}$$

$$= \frac{1000 \times 4 \times 10^{-3}}{4} = 1 \text{ H}$$

$$= 1 \text{ H}$$

6. Option (1) is correct.

$$2t = \left[\frac{2m+1}{2}\right]\frac{\lambda}{n}$$



$$t_{min} = \frac{\lambda}{4n} \qquad [\text{by putting } m = 0]$$

$$= \frac{5.5 \times 10^{-7}}{4 \times 1.38} = 99.6 \text{ nm}$$

7. Option (4) is correct.

$$\vec{M} = i\vec{A}$$

$$= 5 \times (0.2) \times (0.1)\left(-\hat{i}\right)$$

$$= 0.1\left(-\hat{i}\right)$$

$$\vec{\tau} = \vec{M} \times \vec{B}$$

$$= 0.1\left(-\hat{i}\right) \times (2 \times 10^{-3})\left(\hat{j}\right)$$

$$= 2 \times 10^{-4}\left(-\hat{k}\right) \text{ N} - m$$

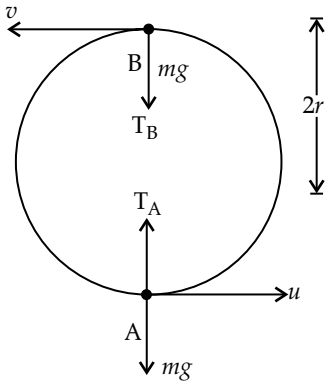
8. Option (3) is correct.

At lowest point,

$$T_A - mg = \frac{mu^2}{r} \qquad ...(1)$$

At highest point,

$$T_B + mg = \frac{mv^2}{r} \qquad ...(2)$$



Gain in PE from from A to B

$$= 2\,mgr = \text{loss in KE}$$

$$\Rightarrow \quad 2mgr = \frac{1}{2}m\ (u^2 - v^2)$$

$$\Rightarrow \quad u^2 - v^2 = 4\ gr \qquad ...(3)$$

from (1), (2), and (3)

$$T_A - T_B - 2mg = \frac{m}{r}\ (u^2 - v^2)$$

$$T_A - T_B = 2mg + \frac{m}{r} \times 4gr = 6\ mg$$

9. Option (3) is correct.

$$\frac{I_{max}}{I_{min}} = \frac{9}{1} = \frac{(a_1 + a_2)^2}{(a_1 - a_2)^2}$$

$$\Rightarrow \quad \frac{a_1 + a_2}{a_1 - a_2} = \frac{3}{1}$$

$$\Rightarrow \quad a_1 + a_2 = 3a_1 - 3a_2$$

$$-4a_2 = -2a_1$$

$$\frac{a_1}{a_2} = \frac{2}{1}$$

10. Option (3) is correct.

$$Y' = \overline{A + B}$$

$$Y = \overline{Y'}$$

$$Y = \overline{\overline{A + B}}$$

$Y = A + B$ which is an OR-gate

11. Option (3) is correct.

M.I. of the uniform circular disc about its diameter = I

M.I. of the disc about its axis = $\frac{1}{2}mr^2$

According to the Theorem of perpendicular axis,

$$I_z = I_x + I_y$$

M.I. of the disc about its axis,

$$I_z = I + I = 2I$$

Thus, $2I = \frac{1}{2}mr^2$

$\therefore \quad mr^2 = 4I$

As per the parallel axes theorem, moment of inertia of disc about the perpendicular axis passing through its rim:

$$= \frac{1}{2}mr^2 + mr^2$$

$$= 2I + 4I = 6I$$

12. Option (3) is correct.

The Equivalence Principle, which asserts that the mass responsible for an object's resistance to acceleration (inertial mass) is identical to the mass that determines the strength of its gravitational attraction (gravitational mass). So, the ratio of the inertial mass to gravitational mass is equal to 1.

13. Option (2) is correct.

Let E_K, E_L, E_M, E_N be the binding energies of K, L, M and N shell. Let E_P be the energy of the incident photon, then

$$E_P - E_K = 24 \text{ keV} \qquad ...(1)$$

$$E_P - E_L = 100 \text{ keV} \qquad ...(2)$$

$$E_P - E_M = 110 \text{ keV} \qquad ...(3)$$

From equation (1) and (2)

$$E(K_\alpha) = E_K - E_L$$

$$= 100 - 24 = 76 \text{ keV}$$

14. Option (3) is correct.

$$^{200}X \longrightarrow {}^{120}Y + {}^{80}Z$$

Energy released = Nuclear energy of products – Nuclear energy of reactants

Energy released = $80 \times 7 + 120 \times 8 - 200 \times 6.5$

= 220 MeV

15. Option (1) is correct.

Let the time period depend on mass (m), amplitude (a) and constant (k) as:

$$T \propto m^\alpha a^\beta k^\gamma$$

$$[M^\circ L^\circ T] \propto M^\alpha L^\beta \left[\frac{ML^2T^{-2}}{L^3}\right]^\gamma$$

$$[M^\circ L^\circ T] \propto M^{\alpha+\gamma}\ L^{\beta-\gamma} T^{-2\gamma}$$

Equating dimensions on both sides

$$-2\gamma = 1$$

$$\Rightarrow \quad \gamma = -\frac{1}{2}$$

$$\alpha + \gamma = 0$$

$$\Rightarrow \quad \alpha = -\gamma$$

$$\alpha = \frac{1}{2}$$

$$\beta - \gamma = 0$$

$$\Rightarrow \quad \beta = \gamma$$

$$\beta = -\frac{1}{2}$$

$$T \propto m^{1/2}\ a^{-1/2}\ k^{-1/2}$$

$$T \propto \sqrt{\frac{m}{ak}}$$

$$T \propto \frac{1}{\sqrt{a}}$$

16. Option (1) is correct.

$$F - 2T = 6a$$

and $T = 4 \times 2a$

$\therefore \quad F - 16\ a = 6a$

$$\Rightarrow \quad a = \frac{F}{22} \quad \Rightarrow \quad a = 1 \text{ m/s}^2$$

$$\therefore \quad a_{CM} = \frac{6 \times 1 + 4 \times 2}{10} = 1.4 \text{ m/s}^2$$

17. Option (2) is correct.

Maximum power transfer theorem:

Internal Resistance = External Resistance

$$2 = (4 + 2) \,||\, x.$$

$$\Rightarrow \quad 2 = \frac{6x}{6+x}$$

$$\Rightarrow \quad 12 + 2x = 6x$$

$$\therefore \quad x = 3\ \Omega$$

18. Option (4) is correct.

It is given in the problem that

$$V_R = 8V$$

$$V_C = 12V$$

and $V_L = 6V$

In a serises LCR circuit, the applied voltage can be given as

$$V = \sqrt{V_R^2 + (V_C - V_L)^2}$$

Now $V = \sqrt{8^2 + (12-6)^2}$

$$\Rightarrow \quad V = \sqrt{64+36}$$

$$\Rightarrow \quad V = \sqrt{100}$$

$$\Rightarrow \quad V = 10V$$

Now $\cos\phi = \frac{R}{Z}$

$$\Rightarrow \quad \cos\phi = \frac{IR}{IZ}$$

$$\Rightarrow \quad \cos\phi = \frac{V_R}{V}$$

Now putting the values of V_R and V in the equation,

$$\Rightarrow \quad \cos\phi = \frac{8}{10}$$

$$\Rightarrow \quad \cos\phi = \frac{4}{5}$$

Also, $V_L = V_C$

So, We get $I(X_L) < (X_C)$

$$\Rightarrow \quad X_L = X_C$$

Hence, the correct answer is option (4).

19. Option (2) is correct.

$$F = \frac{kq^2}{R^2} = \frac{mv^2}{R}$$

$$\Rightarrow \quad v^2 \propto \frac{1}{R}$$

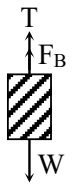
$$\left(\frac{2\pi R}{T}\right)^2 \propto \frac{1}{R}$$

$$\frac{R^2}{T^2} \propto \frac{1}{R}$$

$$R^3 \propto T^2$$

or $T \propto R^{3/2}$.

20. Option (1) is correct.



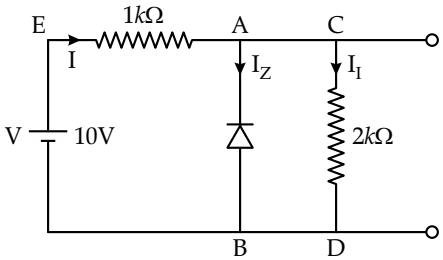
$$T + F_B - W = 0$$

$$W = T + F_B$$

$$= 2.54 \times 9.8 + 10^3 \times 2.197 \times 10^{-3} \times 9.8$$

$$W = 46.43\ N$$

21. Correct answer is [5.5].



Given, $V_Z = 3V$

Let Potential at B = 0V

Potential at E = 10V

Now, $V_C = V_A = 3V$

From the figure

$$I_Z + I_1 = I$$

$$I = \frac{10-3}{1,000} = \frac{7}{1,000}A$$

$$I_1 = \frac{3}{2,000}A$$

Therefore, $I_Z = \frac{7-1.5}{1,000} = 5.5\ mA$

22. Correct answer is [205.00].

$$\text{Work done } W = \frac{1}{2}(40-20)(2-1)$$

$$= 10\ \text{Atm} - \text{L} = 1000\ \text{J}.$$

In cyclic process,

$$\Delta W = 0.$$

$\therefore$ As per the first law of thermodynamics,

$$\Delta Q = W = 1000\ J.$$

$\therefore$ Heat rejected = 1205 – 1000 = 205 J.

23. Correct answer is [2.00].

From conservation of momentum

$$P_i = P_f$$

$$1(2) + 1(0) = (1+1)v$$

$$2 = 2v$$

$$v = 1\ \text{m/s}$$

Now, the initial KE,

$$K_i = \frac{1}{2}(1)(2)^2 = 2J$$

and the final KE,

$$K_f = \frac{1}{2}(2)(1)^2 = 1J$$

$$\text{Loss in K.E.} = (2-1)\text{J} = 1\text{J}$$
$$= \text{Gain in PE}$$
$$1\text{J} = \frac{1}{2}kx^2$$
$$1 = \frac{1}{2}\times 50\times x^2$$
$$x = \frac{1}{5} = 0.2 = \frac{2}{10}\text{m}$$
$$x = 2.$$

24. Correct answers is [1.20].

$$\text{W by electric field} = -5\times 10^{-8}\text{ J}$$
$$\text{W against electric field} = +5\times 10^{-8}\text{ J}$$
$$2\times 10^{-6}\,[2\times a\times (0.1)^2 - 0] = +5\times 10^{-5}\text{ J}$$
$$4\times 10^{-6}\times a\times \frac{1}{100} = 5\times 10^{-8}$$
$$4\times 10^{-8}\times a = 5\times 10^{-8}$$
$$a = \frac{5}{3} = 1.2\text{ V/m}^2$$

25. Correct answer is [4.68].

At equilibrium

$$2T\sin\theta = mg$$
$$\Rightarrow \quad 2.\left(\frac{YA}{a}\right)x\sin\theta.\sin\theta = mg$$
$$\Rightarrow \quad \frac{2YA}{a}\,x.\,\frac{x^2}{a^2} = mg$$
$$\Rightarrow \quad x = \left\{\frac{a^3 mg}{2YA}\right\}^{\frac{1}{3}}$$
$$= \left\{\frac{1\text{ m}\times 5\text{ kg}\times 10\text{ m/s}^2}{2(2.4\times 10^9\text{ N/m}^2)\times 10^{-4}\text{ m}^2}\right\}^{\frac{1}{3}}$$
$$= 4.68\text{ cm}$$

Chemistry

26. Option (1) is correct.

Covalent radius of Atom $= \frac{1}{2}\times$ distance between two nuclei

$$r_{Br_2} = \frac{3.2}{2} = 1.6\,\text{Å}$$

27. Option (4) is correct.

$PCl_5 = sp^3d$, no lone pairs, Trigonal pyramidal
$F_2O = sp^3$, two lone pairs, V-shape
$BCl_3 = sp^2$, no lone pairs, Triangular planar
$NH_3 = sp^3$, one lone pair, Triangular pyramidal

28. Option (1) is correct.

Alkaline solution of Ni^{2+} reacts with dimethyl glyoxime to form red precipitate of nickel dimethylglyoxime.

$$2DMG + NiCl_2 + 2NH_4OH \rightarrow \underset{\text{(Red ppt)}}{[Ni(DMG)_2]\downarrow} + 2NH_4Cl + 2H_2O$$

29. Option (3) is correct.

$$W = -2.303\,nRT\log\frac{V_2}{V_1}$$
$$= -2.303\times 2\times 2\times 300\log\frac{50}{15}$$
$$= -2763.6\,(\log 10 - \log 3)$$
$$= -2763.6\,(1 - 0.4771)$$
$$= -2763.6\times 0.5229$$
$$= -1445\text{ cal.}$$

30. Option (2) is correct.

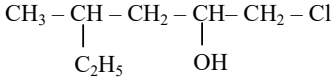
$$Bi_2S_3 \rightleftharpoons \underset{2s}{2Bi^{3+}} + \underset{3s}{3S^{2-}}$$
$$K_{sp} = (2s)^2\,(3s)^3 = 108\,s^5$$
$$108\times 10^{-75} = 108\,s^5$$
$$s = 1.0\times 10^{-15}\text{ mol/L}$$

31. Option (3) is correct.

For weak acid,

$$[H^+] = \sqrt{K_a\cdot C}$$
$$= \sqrt{0.1\times 2\times 10^{-5}}$$
$$= \sqrt{2\times 10^{-6}}$$
$$= \sqrt{2}\times 10^{-3}$$
$$pH = -\log[H^+]$$
$$= -\log\left(\sqrt{2}\times 10^{-3}\right)$$
$$pH = 3 - \log(1.414) \quad (\log 1.414 = 0.150)$$
$$= 2.85$$

32. Option (1) is correct.



1-Chloro-4 methyl hexan-2-ol

33. Option (1) is correct.

Statement I is correct. In asymmetrical molecule, the number of isomers can be calculated by using the formula $= 2^n$.

Where n = no. of double bond showing geometrical isomer.

Statement II is correct. To show geometrical isomerism, molecule must be plannar and have restricted rotation.

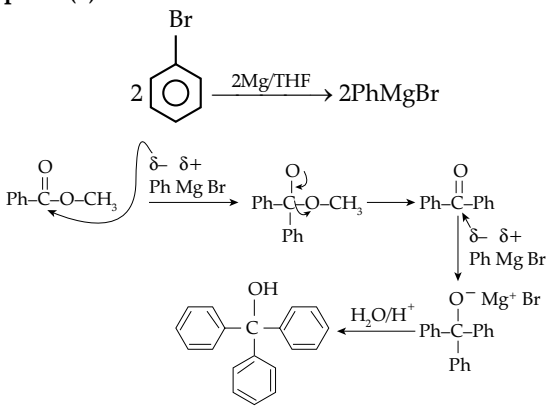
34. Option (1) is correct.

π–bond present in alkenes is weaker than the σ–bond present in alkanes, which makes alkenes less stable. Therefore, Statement I is correct. A carbon–carbon double bond is stronger than a carbon–carbon single bond because more energy

is required to break a sigma and pi bond than to break a sigma bond only. Therefore, statement II is also correct.

C≡C	120 pm
C=C	134 pm
C–C	154 pm

35. Option (2) is correct.



36. Option (2) is correct.

Cerium exists in two different oxidation state +3, +4

It shows Ce^{+4} and acts as a strong oxidising agent and accepts electron. Most stable and common oxidation state of lanthanoids is +3 because of the half filled *d* subshell.

37. Option (3) is correct.

$$[HCOOH] = 0.5 \text{ mol/L}$$

$\Rightarrow$ $(0.5\text{ml} \times 1.05\,\text{m/mL})$ HCOOH in 1 L

$\Rightarrow$ 0.525 g HCOOH in 1 L

$$m = \frac{(0.525/46)}{1 \text{ kg}} \text{ mol}$$

[Assuming dilute solution]

$$\therefore \quad \Delta T_f = iK_f m \Rightarrow i = \frac{\Delta T_f}{K_f m} = \frac{0.0405 \times 46}{1.86 \times 0.525} = 1.9$$

38. Option (2) is correct.

These are two main conditons for hyper conjugation:

(i) The presence of a hydrogen atom (C–H) adjacent to a sp^2 hybrid carbon (Carbocation).

(ii) The presence of a lone pair adjacent to a sp^2 hybrid carbon.

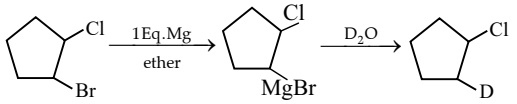
39. Option (3) is correct.

$$R - C \equiv CH \xrightarrow{Na/liq\,NH_3} R - C \equiv \bar{C}\ Na^+$$

$$\xrightarrow{CH_3Cl} R - C \equiv C - CH_3$$

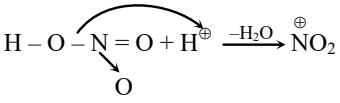
The end product of the reaction will be higher alkyne having one carbon more than 1-alkyne.

40. Option (1) is correct.



41. Option (3) is correct.

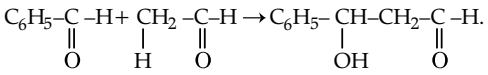
$$H_2SO_4 \rightarrow H^+ + HSO_4^-$$



42. Option (3) is correct.

Ethanol gives the iodoform test (heating with iodine and alkali) but methanol does not give this test.

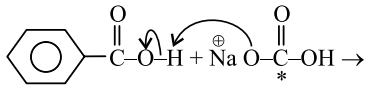
43. Option (4) is correct.



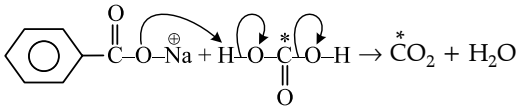
Aldol

In this reaction, only one kind of cross aldol is formed, because benzaldehyde does not have α – H.

44. Option (1) is correct.



Sod. bicarbonate



45. Option (3) is correct.

$C_2H_5NH_2 + HI \rightarrow$ C_2H_5NH–I

Ethyl ammonium iodide

$(CH_3)_3N + HBr \rightarrow$ $(CH_3)_3\overset{+}{N}HBr^-$

trimethyl ammonium bromide

46. Correct answer is [15.00].

Given,

$t_{1/2}$ of A = 5 min $\Rightarrow K = \lambda v^{2/5}$

$t_{1/2}$ of B = 15 min $\Rightarrow K = \lambda v^{2/15}$

Also, [A] = 4 [B]

For first order reaction;

$$\log R_o = \log R_t + kt$$

or $$R_t = R_o\, e^{-kt}$$

According to the question,

$$R_o\,(A) = 4\, R_o\,(B)$$

$$R_{A_t} = R_{B_t}$$

$$\frac{R_{A_{(t)}}}{R_{B_{(t)}}} = \frac{R_o(A) \times e^{-kt}}{R_o(B) \times e^{-kt}}$$

$$\frac{R_{A_{(t)}}}{R_{A_{(t)}}} = \frac{4R_{o_{(B)}} \times e^{-k_A t}}{R_{o_{(B)}} \times e^{-k_B t}}$$

$$\Rightarrow \quad 4 = e\left[\frac{\ln 2}{5} - \frac{\ln 2}{15}\right] \times t$$

Taking ln on both side,

$$\ln 4 = \left[\frac{\ln 2}{5} - \frac{\ln 2}{15}\right] t$$

$$\ln (2)^2 = \left[\frac{\ln 2}{5} - \frac{\ln 2}{15}\right] t$$

$$2 \ln 2 = \left[\frac{\ln 2}{5} - \frac{\ln 2}{15}\right] t$$

$$2 = \frac{2}{15} \times t$$

$$t = 15 \text{ min}$$

47. Correct answer is [0.34].

$$\Delta G^\circ = -nFE^\circ$$

$$E^\circ = \frac{-10.26 \times 1000 \times 4.18}{-2 \times 96500}$$

$$= 0.222$$

$$E = 0.222 - \frac{0.0591}{2} \log \frac{(0.1)^2 (0.1)^2}{1}$$

$$= 0.34 \text{ V}$$

48. Correct answer is [2].

$A = CH_3CHO$

$B = H—\overset{\overset{O}{\|}}{C}—ONa$

$C = CHI_3$

$D = HC \equiv CH$

49. Correct answer is [57].

From $\Delta H = \Delta U + \Delta n_g RT$

$\Delta U = -59.6$ kJ/mol

$T = 27°C = 27 + 273 = 300K$

$H_2F_{2(q)} \longrightarrow H_{2(g)} + F_{2(g)}$

$\Delta n_g = 2 - 1 = 1,$

$$R = 8.314 \times 10^{-3} \frac{kJ}{mol.K}$$

$$\Delta H = -59.6 + 1 \times 8.31 \times 10^{-3} \frac{kJ}{mol\ k} \times 300K$$

$\Delta H = -57.10$ kJ/mol

$\Delta H \approx -57$

50. Correct answer is [8].

(a) The number of unpaired e^- in $[FeF_6]^{3-}$ is 5.

Fe (z = 26) : [Ar] $3d^6$ $4s^2$

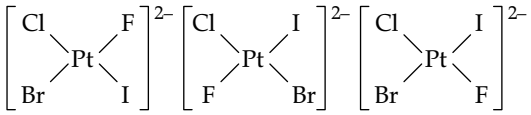
Fe^{3+} : [Ar] $3d^5$ $4s^0$

1	1	1	1	1

$F^\ominus$ ion is a weak field ligand which does not pair the unpaired e^-. So the total number of unpaired e^- = 5.

$$\boxed{x = 5}$$

(b) The number of geometrical isomers of [Pt Cl Br FI]$^{2-}$ is 3.



Total isomers are 3.

$$\boxed{y = 3}$$

(c) The oxidation state of central atom in Ni $(CO)_4$ is 0.

$$\boxed{z = 0}$$

The value of $x + y + z = 5 + 3 + 0 = 8$

Mathematics

51. Option (2) is correct.

Given, Relation R = {(a, b), (b, c)} on the set {a, b, c}

For symmetric $(b, a), (c, b) \in R$

and for transitive $(a, c) \in R$

Now, relation $R = \{(a, b), (b, c), (b, a), (c, b), (a, c)\}$

Again for symmetric $(c, a) \in R$

and for transitive

$\because \quad (a, b), (b, a) \in R$

$\therefore \quad (a, a) \in R$

and $(b, c), (c, b) \in R$

$\Rightarrow \quad (b, b) \in R$

and $(c, a), (a, c) \in R$

$\Rightarrow \quad (c, c) \in R$

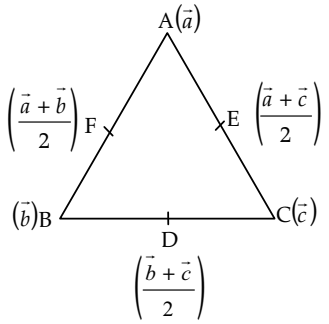
$\therefore$ 7 elements {(b, a), (c, b), (a, c), (c, a), (a, a), (b, b) (c, c)} must be added to the given relation, so that it becomes symmetric and transitive.

Hint:

(1) A relation R on a set A is said to be symmetric relation iff $(a, b) \in R \Rightarrow (b, a) \in R$ for all $a, b, \in A$.

(2) A relation R on a set A is said to be transitive relation iff $(a, b) \in R$, $(b, c) \in R \Rightarrow (a, c) \in R$ for all $a, b, c \in A$.

52. Option (2) is correct.



$\overrightarrow{AD} + \overrightarrow{BE} + \overrightarrow{CF}$

$= \left(\frac{\vec{b}+\vec{c}}{2} - \vec{a}\right) + \left(\frac{\vec{a}+\vec{c}}{2} - \vec{b}\right) + \left(\frac{\vec{a}+\vec{b}}{2} - \vec{c}\right)$

$= \left(\vec{a}+\vec{b}+\vec{c}\right) - \left(\vec{a}+\vec{b}+\vec{c}\right)$

$= 0$

Hence, $|\overrightarrow{AD} + \overrightarrow{BE} + \overrightarrow{CF}| = 0$

53. Option (3) is correct.

Given, $I_m = \int \frac{x^m}{y} dx$

$aI_3 + bI_2 + cI_1 = x^2y$

$\Rightarrow a\int \frac{x^3}{y}dx + b\int \frac{x^2}{y}dx + c\int \frac{x}{y}dx = x^2y$

$\Rightarrow \int \frac{ax^3+bx^2+cx}{y}dx = x^2y$

$\Rightarrow \frac{ax^3+bx^2+cx}{y} = \frac{d}{dx}(x^2y)$

$\Rightarrow \frac{ax^3+bx^2+cx}{y} = 2xy + x^2\frac{dy}{dx}$ ...(1)

Also, $y^2 = 2x^2 + 4x + 3$

$\Rightarrow 2y\frac{dy}{dx} = 4x + 4$

$\Rightarrow \frac{dy}{dx} = \frac{2x+2}{y}$ ...(2)

From eq. (1) and (2)

$\Rightarrow \frac{ax^3+bx^2+cx}{y} = 2xy + \frac{x^2}{y}(2x+2)$

$\Rightarrow ax^3 + bx^2 + cx = 2xy^2 + 2x^3 + 2x^2$

$\Rightarrow ax^3 + bx^2 + cx = 2x(2x^2 + 4x + 3) + 2x^3 + 2x^2$

$\Rightarrow ax^3 + bx^2 + cx = 4x^3 + 8x^2 + 6x + 2x^3 + 2x^2$

$\Rightarrow ax^3 + bx^2 + cx = 6x^3 + 10x^2 + 6x$

On comparing both the sides we get,

$a = 6, b = 10, c = 6$

$\therefore \quad {}^{a+c}P_{b+1} = {}^{12}P_{11} = \frac{12!}{1!} = 12!$

Hint:

(1) Use addition of integration and product rule of differentiation and simplify.

(2) ${}^nP_r = \frac{n!}{(n-r)!}$

54. Option (2) is correct.

Let $I = \int_0^\infty \left[\frac{2}{e^x}\right] dx$

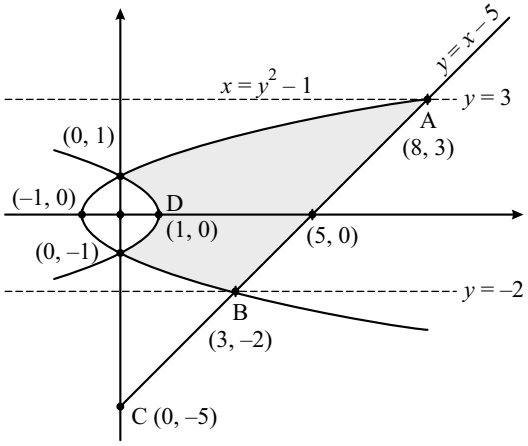
$= \int_0^{\log_e 2} dx + \int_{\log_e 2}^\infty 0 dx$

$= \log_e 2$

55. Option (1) is correct.

Given, Curves $x = |y^2 - 1|$ and $y = x - 5$

$\Rightarrow x = |y^2 - 1| = \begin{cases} -(y^2-1); & -1 < y < 1 \\ y^2 - 1; & y \le -1 \ \& \ y \ge 1 \end{cases}$



Let find intersection points of $y = x - 5$ and $x = y^2 - 1$

$\Rightarrow y + 5 = y^2 - 1$

$\Rightarrow y^2 - y - 6 = 0$

$\Rightarrow (y - 3)(y + 2) = 0$

$\Rightarrow y = 3, -2$

$\Rightarrow$ Intersecting points are (8, 3) and (3, – 2)

$\therefore$ Required area $A = \int_{-2}^{3} \{(y+5) - |y^2 - 1|\} dy$

$\Rightarrow A = \int_{-2}^{3} \{(y+5)dy - \left\{\int_{-2}^{-1}(y^2-1)dy + \int_{-1}^{1} -(y^2-1)dy + \int_{1}^{3}(y^2-1)dy\right\}$

$\Rightarrow A = \left[\frac{y^2}{2} + 5y\right]_{-2}^{3} - \left\{\left[\frac{y^3}{3} - y\right]_{-2}^{-1}\right.$

$$-\left[\frac{y^3}{3}-y\right]_{-1}^{1}+\left[\frac{y^3}{3}-y\right]_{1}^{3}\Bigg\}$$

$$\Rightarrow \text{A} = \left[\frac{9}{2}+15-2+10\right]-\left[-\frac{1}{3}+1+\frac{8}{3}-2\right]$$

$$+\left[\frac{1}{3}-1+\frac{1}{3}-1\right]-\left[\frac{27}{3}-3-\frac{1}{3}+1\right]$$

$$\Rightarrow \text{A} = \frac{39}{2}+8-\frac{4}{3}-\frac{4}{3}-\frac{20}{3}$$

$$\Rightarrow \text{A} = \frac{55}{2}-\frac{28}{3}$$

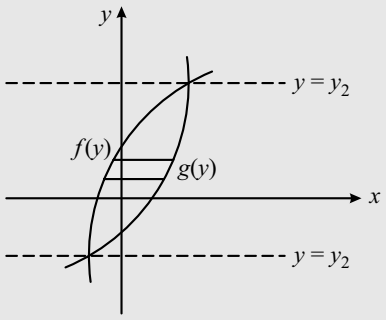
$$\Rightarrow \text{A} = \frac{109}{6} \text{ sq. units}$$

Hint:

(1) Draw the curves using the concept of modulus function and identity the required region.

(2) Area bounded by two curves $x = g(y)$ and $x = f(y)$ such that $g(y) > f(y)$ is given by

$$\Rightarrow \qquad \text{A} = \int_{y_1}^{y_2}[g(y)-f(y)]dy$$



56. Option (3) is correct.

$$\text{I} = \int\sqrt{1+x^{2n}}\left(\frac{\log_e(1+x^{2n})-2n\log_e x}{x^{3n+1}}\right)dx$$

$$\Rightarrow \text{I} = \int\sqrt{\frac{1}{x^{2n}}+1}\left\{\frac{\log_e\left(\frac{1}{x^{2n}}+1\right)}{x^{2n+1}}\right\}dx$$

$$\Rightarrow \text{Let} \qquad \frac{1}{x^{2n}}+1 = u^2$$

$$\Rightarrow \qquad -\frac{2n}{x^{2n+1}}dx = 2u\,du$$

$$\text{So,} \quad \text{I} = -\int\frac{u^2\log_e u^2}{n}du$$

$$\Rightarrow \text{I} = -\frac{2}{n}\int u^2\log_e u\,du$$

$$\Rightarrow \text{I} = -\frac{2}{n}\left[\log_e u\frac{u^3}{3}-\int\left(\frac{1}{u}\right)\frac{u^3}{3}du\right]$$

$$\Rightarrow \text{I} = -\frac{2}{n}\left[\frac{u^3}{3}\log_e u-\frac{1}{3}\,\frac{u^3}{3}\right]+c$$

$$\Rightarrow \text{I} = -\frac{2}{3n}u^3\left[\log_e u-\frac{1}{3}\right]+c$$

$$\Rightarrow \text{I} = -\frac{2}{3n}\left(\frac{1}{x^{2n}}+1\right)^{3/2}$$

$$\left[\log_e\left(1+\frac{1}{x^{2n}}\right)^{1/2}-\frac{1}{3}\right]+c$$

$$\Rightarrow \text{I} = -\frac{2}{3n}\frac{\sqrt{(1+x^{2n})^3}}{x^{3n}}$$

$$\left[\frac{1}{2}\log_e(1+x^{2n})-\frac{1}{2}\log_e x^{2n}-\frac{1}{3}\right]+c$$

Hint:

(1) Simplify given integral and substitute $\frac{1}{x^{2n}}+1=u^2$ and solved further using integration by parts method.

(2) $$\int f(x)g(x)dx = f(x)\int g(x)dx$$

$$-\int\left[f'(x)\left\{\int g(x)\,dx\right\}\right]dx$$

57. Option (2) is correct.

Given, $\quad y = a\log_e|x| + bx^2 + x$

$$\Rightarrow \qquad y = \begin{cases} a\log_e x+bx^2+x;\ x>0 \\ a\log_e(-x)+bx^2+x;\ x<0 \end{cases}$$

$$\Rightarrow \qquad \frac{dy}{dx} = \begin{cases} \frac{a}{x}+2bx+1,\ x>0 \\ \frac{a}{x}+2bx+1,\ x<0 \end{cases}$$

Since, $y = f(x)$ has it's extremum values at $x = -1$ and $x = 2$

$$\therefore \left(\frac{dy}{dx}\right)_{x=-1}=0 \text{ and } \left(\frac{dy}{dx}\right)_{x=2}=0$$

$\Rightarrow -a - 2b + 1 = 0$ and $\frac{a}{2} + 4b + 1 = 0$

$\Rightarrow \qquad 2b + a = 1$ and $a + 8b = -2$

$\Rightarrow \qquad a = 2$ and $b = -\frac{1}{2}$

58. Option (3) is correct.

$$\frac{dy}{dx} = \frac{a\cos\theta}{-a\sin\theta+\dfrac{a\sec^2\dfrac{\theta}{2}}{\tan\dfrac{\theta}{2}}\times\dfrac{1}{2}}$$

$$= \frac{a\cos\theta}{-a\sin\theta + \dfrac{a}{2\cos\dfrac{\theta}{2}\sin\dfrac{\theta}{2}}}$$

$$= \frac{a\cos\theta}{\dfrac{a}{\sin\theta} - a\sin\theta}$$

$$= \frac{a\cos\theta\sin\theta}{a - a\sin^2\theta}$$

$$= \frac{\cos\theta\sin\theta}{\cos^2\theta}$$

$$= \tan\theta$$

59. Option (3) is correct.

As we know,

$$0 \le \sin^2 x \le 1$$

$$\Rightarrow \quad 0 \le 16\sin^2 x \le 16$$

$$\Rightarrow \quad 1 \le 16\sin^2 x + 1 \le 17$$

$$\Rightarrow \quad \log_2(1) \le \log_2(16\sin^2 x + 1) \le \log_2(17)$$

$$\Rightarrow \quad 0 \le \log_2(16\sin^2 x + 1) \le \log_2(17)$$

$$\Rightarrow \quad 0 \ge -\log_2(16\sin^2 x + 1) \ge -\log_2(17)$$

$$\Rightarrow \quad 2 \ge 2 - \log_2(16\sin^2 x + 1) \ge 2 - \log_2(17)$$

As logarithm of negative value is not defined so we will consider

$$2 \ge 2 - \log_2(16\sin^2 x + 1) > 0$$

$$\Rightarrow \log_{\sqrt{2}} 2 =$$

$$\log_{\sqrt{2}}(2 - \log_2(16\sin^2 x + 1)) > \log\sqrt{2}$$

$$\Rightarrow \log_{\sqrt{2}}(\sqrt{2})^2 = \log_{\sqrt{2}}(2 - \log_2(16\sin^2 x + 1)) > -\infty$$

$$\Rightarrow \quad 2 \ge \log_{\sqrt{2}}(2 - \log_2(16\sin^2 x + 1)) > -\infty$$

Range of given function is $(-\infty, 2]$

$$\Rightarrow \quad P = 2$$

Hint: Use $0 \le \sin^2 x \le 1$ and then use properties of inequality and logarithm to solve further.

60. Option (2) is correct.

Given, $\lim\limits_{x\to 1} \sec^{-1}\left(\dfrac{k^2}{\log_e x} - \dfrac{k^2}{x-1}\right)$ exists.

Let us consider

$$= \lim_{x\to 1}\left(\frac{k^2}{\log_e x} - \frac{k^2}{x-1}\right)$$

$$= \lim_{x\to 1} k^2\left(\frac{1}{\log_e x} - \frac{1}{x-1}\right)$$

$$= \lim_{x\to 1} k^2\left(\frac{(x-1) - \log_e x}{(\log_e x)(x-1)}\right)$$

$$= \lim_{x\to 1} k^2\left(\frac{(x-1) - \log_e\{1 + (x-1)\}}{(x-1)\log_e\{1 + (x-1)\}}\right)$$

$$\lim_{x\to 1} k^2\left(\frac{(x-1) - (x-1) + \dfrac{(x-1)^2}{2} - \dfrac{(x-1)^3}{3} + \ldots.}{(x-1)\left\{(x-1) - \dfrac{(x-1)^2}{2} + \dfrac{(x-1)^3}{3} + \ldots..\right\}}\right)$$

$$= \lim_{x\to 1} k^2\left(\frac{(x-1)^2\left\{\dfrac{1}{2} - \dfrac{(x-1)}{3} + \ldots..\right\}}{(x-1)^2\left\{1 - \dfrac{(x-1)}{2} + \dfrac{(x-1)^2}{3}\ldots..\right\}}\right)$$

$$= \lim_{x\to 1} k^2\left(\frac{\dfrac{1}{2} - \dfrac{(x-1)}{3} + \ldots..}{1 - \dfrac{(x-1)}{2} + \ldots..}\right)$$

$$\Rightarrow \quad = \frac{k^2}{2}$$

Now, given that $\lim\limits_{x\to 1} \sec^{-1}\left(\dfrac{k^2}{2}\right)$ exists.

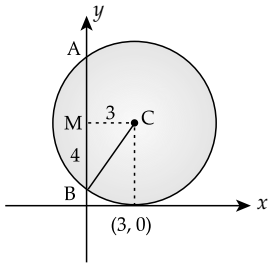
$$\Rightarrow \quad \frac{k^2}{2} \ge 1$$

$\Rightarrow k^2$ has smallest value as 2.

Hint: Use $\log_e(x+1) = x - \dfrac{x^2}{2} + \dfrac{x^3}{3} - \ldots..$ and simplify.

61. Option (1) is correct.

Let centre of circle is C and circle cuts the y-axis at B and A. Let mid-point of chord BA is M.



$$CB = \sqrt{MC^2 + MB^2}$$

$$\Rightarrow \quad CB = \sqrt{3^2 + 4^2}$$

$$= 5 = \text{radius of circle}$$

$\therefore$ Equation of circle is,

$(x-3)^2 + (y-5)^2 = 5^2$

(3, 10) satisfies this equation.

Although, there will be another circle satisfying the same conditions that will lie below the x-axis having equation

$(x-3)^2 + (y+5)^2 = 5^2$

62. Option (1) is correct.

Given, $\sum_{i=1}^{8}(x_i - 7) = 8$

and $\sum_{i=1}^{8}(x_i - 7)^2 = 40$

Let $X_i = x_i - 7$

Now, $\bar{X} = \frac{1}{8}\sum_{i=1}^{8} X_i$

$\Rightarrow \bar{X} = \frac{1}{8}\sum_{i=1}^{8} x_i - 7$

$\Rightarrow \bar{X} = \frac{1}{8} \times 8 = 1$

Now, Var (X) = $\sigma^2 = \frac{1}{N}\sum x^2 - (\bar{X})^2$

$= \frac{1}{8}\sum (x_i - 7)^2 - (1)^2$

$= \frac{1}{8} \times 40 - 1$

$= 4$

$\because$ S.D. $= \sqrt{\text{Var (X)}}$

$\Rightarrow$ S.D. $= \sqrt{4} = 2$

As we know by property of S.D., it is independent of change of origin.

So, standard deviations of $x_1, x_2, x_3, \ldots\ldots x_8$ is 2.

So, both statements are true and statement 2 is correct explanation for statement-1.

Hint:

(1) Use Var $(x) = \sigma^2 = \frac{1}{N}\sum x^2 - (\bar{x})^2$

(2) S.D. $= \sqrt{\text{Var}(x)}$

(3) Recall the properties of standard deviation.

63. Option (3) is correct.

Given, A (1, 6, 3)

AB is perpendicular to

$$x = \frac{y-1}{2} = \frac{z-2}{3}$$

Let $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3} = \lambda$

$\Rightarrow$ Coordinates of B are $(\lambda, 2\lambda + 1, 3\lambda + 2)$

Now direction ratios of AB are $\lambda - 1, 2\lambda + 1 - 6, 3\lambda + 2 - 3$

$\Rightarrow$ D.Rs of AB are $\lambda - 1, 2\lambda - 5, 3\lambda - 1$

Also, D.Rs of the given line $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$ are 1, 2, 3.

Now, AB is perpendicular to the given line.

$\Rightarrow (\lambda - 1)1 + (2\lambda - 5)2 + (3\lambda - 1)3 = 0$

$\Rightarrow \lambda - 1 + 4\lambda - 10 + 9\lambda - 3 = 0$

$\Rightarrow 14\lambda - 14 = 0$

$\Rightarrow \lambda = 1$

$\Rightarrow$ Coordinates of B are (1, 2(1) + 1, 3(1) + 2)

i.e., (1, 3, 5)

Let the image of point A (1, 6, 3) be A′ (x_1, y_1, z_1)

$\Rightarrow$ B will be the mid point of AA′

$\therefore$ By mid point formula,

$$\frac{x_1 + 1}{2} = 1$$

$$\frac{y_1 + 6}{2} = 3$$

$$\frac{z_1 + 3}{2} = 5$$

$\Rightarrow x_1 = 2 - 1$

$\Rightarrow x_1 = 1$

$\Rightarrow y_1 = 6 - 6$

$\Rightarrow y_1 = 0$

$\Rightarrow z_1 = 10 - 3$

$\Rightarrow z_1 = 7$

So, image of point A is (1, 0, 7) w.r.t. the given line.

As we know, projection of line segment joining P (x, y, z) and Q (x_2, y_2, z_2) on a line having direction cosines l, m, n is.

$|l(x_2 - x_1) + m(y_2 - y_1) + n(z_2 - z_1)|$

$\therefore$ Projection of AB with A (1, 6, 3) and B (1, 3, 5) on line having direction cosines $\frac{1}{\sqrt{2}}, 0, -\frac{1}{\sqrt{2}}$ is

$$\Rightarrow \left|\frac{1}{\sqrt{2}}(1-1) + 0(6-3) + \left(-\frac{1}{\sqrt{2}}\right)(3-5)\right|$$

$$= \left|0 + 0 + \sqrt{2}\right|$$

$$= \sqrt{2}$$

Also, by distance formula

$$\text{AB} = \sqrt{(1-1)^2 + (6-3)^2 + (3-5)^2}$$

$\Rightarrow \text{AB} = \sqrt{0 + 9 + 4}$

$\Rightarrow \text{AB} = \sqrt{13}$

Hint:

(1) If direction ratios of two perpendicular lines are a_1, b_1, c_1 and a_2, b_2, c_2 then $a_1a_2 + b_1b_2 + c_1c_2 = 0$.

(2) The projection of line segment joining P (x_1, y_1, z_1) and (x_2, y_2, z_2) on a line having direction cosines l, m, n is

$|l(x_2 - x_1) + m(y_2 - y_1) + n(z_2 - z_1)|$

64. Option (4) is correct.

Equation of ellipse

$$\frac{x^2}{16} + \frac{y^2}{7} = 1 \quad \text{....(i)}$$

And equation of hyperbola

$$\frac{x^2}{144} - \frac{y^2}{\alpha} = \frac{1}{25} \quad \text{....(ii)}$$

$$\Rightarrow \quad \frac{x^2}{\frac{144}{25}} - \frac{y^2}{\frac{\alpha}{25}} = 1$$

Have same foci, then

$$a^2 - b^2 = l^2 + m^2$$

$$\Rightarrow \quad 16 - 7 = \frac{144}{25} + \frac{\alpha}{25}$$

$$\Rightarrow \quad 9 = \frac{144}{25} + \frac{\alpha}{25}$$

$$\Rightarrow \quad 9 \times 25 = 144 + \alpha$$

$$\Rightarrow \quad \alpha = 225 - 144 = 81$$

$$\therefore \Rightarrow \quad \alpha = 81$$

Latus Rectum of hyperbola

$$= \frac{2b^2}{a} = \frac{2\times\frac{\alpha}{25}}{\frac{12}{5}}$$

$$= \frac{2\times 81}{25}\times\frac{5}{12} = \frac{27}{10}$$

65. Option (3) is correct.

Let roots be α and β then from equation

$$-x^2 - bx + a = 0$$

We have, $\alpha\beta = -a < 0$

$\Rightarrow$ Both the roots are in opposite sign and

$$\alpha + \beta = -b < 0$$

$\Rightarrow$ Both roots are in opposite sign and greater root in magnitude is negative.

66. Option (2) is correct.

Given, system of linear equations

$$-x + y + 2z = 0$$
$$3x - ay + 5z = 1$$
$$2x - 2y - az = 7$$

$$\Delta = \begin{vmatrix} -1 & 1 & 2 \\ 3 & -a & 5 \\ 2 & -2 & -a \end{vmatrix}$$

$\Rightarrow \quad \Delta = -1(a^2 + 10) - 1(-3a - 10) + 2(-6 + 2a)$

$\Rightarrow \quad \Delta = -a^2 - 10 + 3a + 10 - 12 + 4a$

$\Rightarrow \quad \Delta = -a^2 + 7a - 12$

$\Rightarrow \quad \Delta = -(a - 3)(a - 4)$

Also, $\Delta_1 = \begin{vmatrix} 0 & 1 & 2 \\ 1 & -a & 5 \\ 7 & -2 & -a \end{vmatrix}$

$\Rightarrow \quad \Delta_1 = 0 - 1(-a - 35) + 2(-2 + 7a)$

$\Rightarrow \quad \Delta_1 = a + 35 - 4 + 14a$

$\Rightarrow \quad \Delta_1 = 15a + 31$

For an inconsistent system $\Delta = 0$ and $\Delta_1 \neq 0$

$\Rightarrow \quad -(a - 3)(a - 4) = 0$

$\Rightarrow \quad a = 3$ or 4

and for $a = 3$, $\Delta_1 = 45 + 31 = 76 \neq 0$.

for $a = 4$, $\Delta_1 = 60 + 31 = 91 \neq 0$.

$\Rightarrow \quad P = \{3, 4\}$

$\Rightarrow \quad \Sigma P = 7$

For infinitely many solutions, $\Delta = 0$ and $\Delta_1 = \Delta_2 = \Delta_3 = 0$.

$\Rightarrow \quad -(a - 3)(a - 4) = 0$

$\Rightarrow \quad a = 3$ or 4

for $a = 3$, $\Delta_1 = 76 \neq 0$

for $a = 4$, $\Delta_1 = 91 \neq 0$

So, no values of a possible

$\Rightarrow \quad Q = \{\phi\}$

$\Rightarrow \quad \Sigma Q = 0$

$\Rightarrow \quad \Sigma P + \Sigma Q = 7 + 0 = 7.$

Hint:

(1) For an inconsistent system $\Delta = 0$ and one of the $\Delta_1, \Delta_2, \Delta_3$ not equals to zero.

(2) For infinitely many solution $\Delta = 0$ and $\Delta_1 = \Delta_2 = \Delta_3 = 0$.

67. Option (1) is correct.

$\because$ Sum of n terms of A.P with first terms a and common difference d is given by

$$S_n = \frac{n}{2}[a + (n-1)d]$$

$$\therefore \quad \frac{S_{n_1}}{S_{n_2}} = \frac{\frac{n}{2}\left[2a_1 + (n-1)d_1\right]}{\frac{n}{2}\left[2b_1 + (n-1)d_2\right]} = \frac{2n}{n+1}$$

$$\Rightarrow \quad \frac{a_1 + \frac{(n-1)}{2}d_1}{b_1 + \frac{(n-1)}{2}d_2} = \frac{2n}{n+1} \quad \text{...(i)}$$

For $T_{8,}$ we know $\frac{n-1}{2} = 7$

$\Rightarrow \quad n = 15$

Put $n = 15$ in (i), we get

$$\frac{(T_8)_1}{(T_8)_2} = \frac{30}{16} = \frac{15}{8}$$

68. Option (2) is correct.

$$T_4 = {}^nC_3(ax)^{n-3}\left(\frac{1}{x}\right)^3$$

$$= {}^nC_3\, a^{n-3}\, x^{n-6} = \frac{5}{2} \text{ (given)}$$

$$\therefore \quad n - 6 = 0$$

$\Rightarrow \quad n = 6$

$\Rightarrow \quad T_4 = {}^6C_3 a^{6-3} = \frac{5}{2}$

$\Rightarrow \quad a^3 = \frac{1}{8} = \left(\frac{1}{2}\right)^3$

$\Rightarrow \quad a = \frac{1}{2}$

$\Rightarrow \quad a = \frac{1}{2},\ n = 6$

69. Option (1) is correct.

$${}^{(n+1)}C_{(n-2)} - {}^{(n+1)}C_{(n-1)} \le 100$$

$$\Rightarrow \frac{(n+1)!}{(n-2)!(n+1-n+2)!} - \frac{(n+1)!}{(n-1)!(n+1-n+1)!} \le 100$$

$$\Rightarrow \frac{(n+1)!}{(n-2)!3!} - \frac{(n+1)!}{(n-1)!2!} \le 100$$

$$\Rightarrow (n+1)!\left[\frac{1}{3!(n-2)!} - \frac{1}{2!(n-1)!}\right] \le 100$$

$$\Rightarrow (n+1)!\left[\frac{(n-1)! - 3(n-2)!}{6(n-1)!(n-2)!}\right] \le 100$$

$$\Rightarrow \frac{(n+1)n(n-1)!\left[(n-1)(n-2)! - 3(n-2)!\right]}{6(n-1)!(n-2)!} \le 100$$

$$\Rightarrow \frac{(n+1)\,n\,(n-1)!\,(n-2)!\,(n-1-3)}{6(n-1)!(n-2)!} \le 100$$

$$\Rightarrow \frac{(n+1)\,n\,(n-4)}{6} \le 100$$

$$\Rightarrow n(n+1)(n-4) \le 600$$

Put $n = 1, 2, 3, 4, 5, 6, 7, 8, 9, \ldots\ldots$

For $n = 5$, $600 \ge 5 \times 6 \times 1$ holds good

For $n = 8$, $600 \ge 8 \times 9 \times 4$
$= 288$ holds good

For $n = 9$, $600 \ge 9 \times 10 \times 5$
$= 450$ holds good

For $n = 10$, $600 \le 10 \times 11 \times 6$
$= 650$ does not hold

Hence, the number of positive integer which is satisfying the given inequality must be 9.

Shortcut Method:

$${}^{n+1}C_{n-2} - {}^{n+1}C_{n-1} \le 100$$

$$\Rightarrow \quad {}^{n+1}C_3 - {}^{n+1}C_2 \le 100$$

$$\Rightarrow \quad \frac{(n+1)n(n-1)}{6} - \frac{(n+1)n}{2} \le 100$$

$$\Rightarrow \quad n\,(n+1)\,(n-4) \le 600$$

Consider

$$f(x) = x\,(x+1)\,(x-4)\ ;\ x \ge 1$$
$$= x^3 - 3x^2 - 4x$$

$\Rightarrow \quad f'(x) = 3x^2 - 6x - 4$

If $\quad f'(x) > 0$

$$x \in \left(1 + \frac{\sqrt{21}}{3}, \infty\right)$$

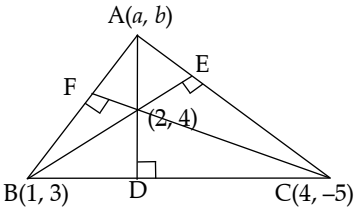
It is easy to check that

$f(10) = 10 \times 11 \times 6 = 660 > 600$

and $\quad f(9) = 9 \times 10 \times 5 = 450 < 600$

$\therefore \quad x = 9$ is the last integer so that $f(x) < 600$.

70. Option (3) is correct.



From the figure

Slope of $BE = 1$

Slope of $AC = -1$

$\because$ (slope of $BE \times$ slope of $AC = -1$)

$\therefore$ Equation AC is $x + y = -1$ ...(i)

Slope of $CF = \frac{-9}{2}$

Slope of $AB = \frac{2}{9}$

$\because$ (slope of $CF \times$ Slope of $AB = -1$)

$\therefore$ Equation AB is $2x - 9y = -25$ ...(ii)

Solving (i) and (ii), we get

$$(a, b) = \left(\frac{-34}{11}, \frac{23}{11}\right)$$

$\therefore$ The value of $33b + 22a = 1$

71. Correct answer is [19].

Given lines L_1: $\frac{x+3}{5} = \frac{y-1}{2} = \frac{z+4}{3} = \lambda$ (let)

L_2: $\frac{x-1}{1} = \frac{y+2}{3} = \frac{z-2}{-2} = \mu$ (let)

and line with direction ratios $\langle 0, 2, -1\rangle$ meets the line L_1 and L_2 at P and Q respectively.

Let the coordinates of P be $(5\lambda - 3, 2\lambda + 1, 3\lambda - 4)$ and coordinates of Q be $(\mu + 1, 3\mu - 2, -2\mu + 2)$

Now, direction ratios of line PQ

$= \mu - 5\lambda + 4, 3\mu - 2\lambda - 3,$
$-2\mu - 3\lambda + 6.$

$\therefore$ direction ratios of line PQ = 0, 2, –1

$\therefore \quad \frac{\mu-5\lambda+4}{0} = \frac{3\mu-2\lambda-3}{2} = \frac{-2\mu-3\lambda+6}{-1} = t$

$\Rightarrow \quad \mu - 5\lambda + 4 = 0$ ...(i)

$3\mu - 2\lambda - 3 = 2t$ ...(ii)

$-2\mu - 3\lambda + 6 = -t$ ...(iii)

From eq. (ii) and eq. (iii),

$3\mu - 2\lambda - 3 - 4\mu - 6\lambda + 12 = 0$

$\Rightarrow \quad -\mu - 8\lambda + 9 = 0$

$\Rightarrow \quad \mu + 8\lambda = 9$ ...(iv)

On solving eq. (i) and eq. (iv), $\mu = 1$ and $\lambda = 1$

$\therefore$ P = (2, 3, –1) and Q = (2, 1, 0)

Now,

$$\text{area of } \Delta OPQ = \frac{1}{2}\left|\overrightarrow{OP}\times\overrightarrow{OQ}\right|$$

$$\Rightarrow \quad \frac{1}{n}\sqrt{m} = \frac{1}{2}\left|(2\hat{i}+3\hat{j}-\hat{k})\times(2\hat{i}+\hat{j})\right|$$

$$\Rightarrow \quad \frac{1}{n}\sqrt{m} = \frac{1}{2}\left|\hat{i}-2\hat{j}-4\hat{k}\right|$$

$$\Rightarrow \quad \frac{1}{n}\sqrt{m} = \frac{1}{2}\sqrt{1+4+16}$$

$$\Rightarrow \quad \frac{1}{n}\sqrt{m} = \frac{1}{2}\sqrt{21}$$

$\Rightarrow n = 2$ and $m = 21$

$\Rightarrow \quad m - n = 19$

Hint:

(1) Assume coordinates of P and Q in parametric form and find direction ratios of PQ and compare it with given direction ratios of line and solved further.

(2) Area of triangle OPQ $= \frac{1}{2}\left|\overrightarrow{OP}\times\overrightarrow{OQ}\right|$

72. Correct answer is [17].

Required probability = when no machine has fault + when only one machine has fault + when only two machines have fault.

$$= {}^5C_0\left(\frac{3}{4}\right)^5 + {}^5C_1\left(\frac{1}{4}\right)\left(\frac{3}{4}\right)^4 + {}^5C_2\left(\frac{1}{4}\right)^2\left(\frac{3}{4}\right)^3$$

$$= \frac{243}{1024}+\frac{405}{1024}+\frac{270}{1024}$$

$$= \frac{918}{1024} = \frac{459}{512}$$

$$= \frac{27\times17}{64\times8}$$

$$\Rightarrow \quad \left(\frac{3}{4}\right)^3 \times k = \left(\frac{3}{4}\right)^3 \times \frac{17}{8}$$

$$\therefore \quad k = \frac{17}{8}$$

So, $8k = 17$

73. Correct answer is [78].

Given, $m = \int_{-1}^{10} \text{sgn}(x-[x])dx$

$\Rightarrow \quad m = \int_{-1}^{10} \text{sgn}\{x\}dx$ $\{\because x = [x] + \{x\}\}$

$\Rightarrow \quad m = 11\int_0^1 \text{sgn}(x)dx$ $\{\because$ Period of $\{x\}$ is 1$\}$

$\Rightarrow \quad m = 11\int_0^1 1\,dx$

$\Rightarrow \quad m = 11[x]_0^1$

$\Rightarrow \quad m = 11$

and $\quad n = \int_0^{26} \frac{e^{\cos\pi\{x\}}}{e^{\cos\pi\{x\}}+e^{-\cos\pi\{x\}}}dx$

$\Rightarrow \quad n = 26\int_0^1 \frac{e^{\cos\pi x}}{e^{\cos\pi x}+e^{-\cos\pi x}}dx$ ...(i)

$\Rightarrow \quad n = 26\int_0^1 \frac{e^{\cos\pi(1-x)}}{e^{\cos\pi(1-x)}+e^{-\cos\pi(1-x)}}dx$

$\left\{\because \int_a^b f(x)dx = \int_a^b f(a+b-x)dx\right\}$

$\Rightarrow \quad n = 26\int_0^1 \frac{e^{-\cos\pi x}}{e^{-\cos\pi x}+e^{\cos\pi x}}dx$ ...(ii)

Adding eq. (i) and eq. (ii), we get,

$2n = 26\int_0^1 1.dx$

$\Rightarrow \quad 2n = 26[x]_0^1$

$\Rightarrow \quad n = 13$

$\therefore \quad {}^nC_m = {}^{13}C_{11} = {}^{13}C_2$

$= \frac{13.12}{2.1} = 78$

Hint:

(1) Use $x = [x] + \{x\}$

(2) $\text{sgn } x = \begin{cases} 1 & ; \quad x > 0 \\ -1 & ; \quad x < 0 \end{cases}$

(3) Use $\int_0^{nT} f(x)dx = n\int_0^T f(x)dx$;

where $f(x)$ is periodic function with period T

(4) $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$

74. Correct answer is [600].

Since, D is a skew symmetric matrix

$\therefore \quad D + O^T = 0$

$\Rightarrow \quad a\alpha^2 + b\alpha + c = 0,$

$a\beta^2 + b\beta + c = 0$

$a\gamma^2 + b\gamma + c = 0$

$\Rightarrow \quad ax^2 + bx + c = 0$ has three roots

$\Rightarrow \quad a = b = c = 0$

$$\therefore \quad \lambda = \begin{vmatrix} 1 & 1 & 2 \\ 3 & 4 & 1 \\ 9 & 0 & 3 \end{vmatrix}$$

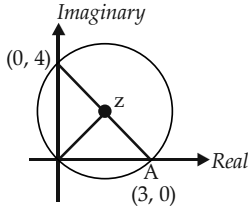
$$= -60$$

$$\Rightarrow \quad |10\lambda| = |10 \times -60| = 600$$

75. Correct answer is [5.00].

z is equidistant from (0, 0), (3, 0) and (0, 4)

$\Rightarrow z$ is at the circumcentre of ΔOAB



$$\Rightarrow \quad |z| = \frac{5}{2}$$

$$\Rightarrow \quad |2z| = 5.00$$

❐❐