Sample Question Paper-5

MATHEMATICS (Code-041)

Class 12

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

Read the following instructions carefully and strictly follow them:

(i) *This Question Paper contains **38** questions. **All** questions are **compulsory**.*

(ii) *Question paper is divided into **FIVE** sections, Section **A, B, C, D** and **E**.*

(iii) *In **Section A:** Questions no. **1** to **18** are Multiple Choice Questions (MCQs) and Questions no. **19** and **20** are Assertion-Reason based questions of 1 mark each.*

(iv) *In **Section B:** Questions no. **21** to **25** are Very Short Answer (VSA) type questions, carrying **2** marks each.*

(v) *In **Section C:** Questions no. **26** to **31** are Short Answer (SA) type questions, carrying **3** marks each.*

(vi) *In **Section D:** Questions no. **32** to **35** are Long Answer (LA) type questions, carrying **5** marks each.*

(vii) *In **Section E:** Questions no. **36** to **38** are case study based questions, carrying **4** marks each.*

(viii) *There is no overall choice. However, an internal choice has been provided in **2** questions in Section-**B**, **3** questions in Section-**C**, **2** questions in Section-**D** and **2** questions in Section-**E**.*

(ix) *Use of calculators is NOT allowed.*

Section-A

This section has 20 multiple choice questions of 1 mark each.

1. Which of the following matrices is non-singular ?

(A) $\begin{bmatrix} -6 & 3 & 5 \\ 0 & 0 & 0 \\ 4 & 9 & -5 \end{bmatrix}$ **(B)** $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ **(C)** $\begin{bmatrix} 1 & 3 & 2 \\ 3 & -2 & 6 \\ 4 & 7 & 8 \end{bmatrix}$ **(D)** $\begin{bmatrix} 0 & 2 & 5 \\ 2 & 0 & 7 \\ 5 & 7 & 0 \end{bmatrix}$ [R]

2. A box B_1 contains 1 white ball and 3 red balls. Another box B_2 contains 2 white balls and 3 red balls. If one ball is drawn at random from each of the boxes B_1 and B_2, then the probability that the two balls drawn are of the same colour is

(A) $\frac{1}{10}$ **(B)** $\frac{9}{10}$ **(C)** $\frac{9}{20}$ **(D)** $\frac{11}{20}$ [Ap]

3. Given two independent events A and B such that $P(A) = 0.3$ and $P(B) = 0.6$, then $P(A' \cup B') = ...$

(A) 0.18 **(B)** 0.09 **(C)** 0.36 **(D)** 0.28 [R]

4. Look at the differential equation given below. What is its degree ? $y^3 = e^{4\frac{d^2y}{dx^2}}$

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 [U]

5. If $R = \{(x, y); x, y \in Z, x^2 + y^2 \leq 4\}$ is a relation is set Z, then domain of R is

(A) $\{0, 1, 2\}$ **(B)** $\{-2, -1, 0, 1, 2\}$ **(C)** $\{0, -1, -2\}$ **(D)** $\{-1, 0, 1\}$ [A]

6. Shown below is a differential equation.

$$\sin x \frac{dy}{dx} + y \cos x = \tan x$$

Which of the following is the integrating factor for the above differential equation?

$e^{\int \sin(x)dx}$ Expression 1 $e^{\int \cos(x)dx}$ Expression 2 $e^{\int \cot(x)dx}$ Expression 3

(A) Expression 1 **(b)** Expression 2 **(C)** Expression 3

(D) The equation has no integrating factor as it is not a Linear differential equation [U]

7. The projection of vector $\vec{a} = 2\hat{i} + 3\hat{j} + 2\hat{k}$ on the vector $\vec{b} = 2\hat{i} + 2\hat{j} + \hat{k}$. is

(A) 12 **(B)** 8 **(C)** 4 **(D)** 0 U

8. Let l_i, m_i, n_i $i = 1, 2, 3$ be the direction cosines of three mutually perpendicular vector in space. Then $AA' =$........

where $A = \begin{bmatrix} l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \\ l_3 & m_3 & n_3 \end{bmatrix}$.

(A) I **(B)** I_2 **(C)** I_3 **(D)** None of these A

9. What is the value of the following integral ?

$\int_{-2}^{2} (2 - |x|)\, dx$

(A) 0 **(B)** 4 **(C)** 8 **(D)** 12 U+R

10. If A and B are two independent events with $P(A) = \frac{1}{3}$ and $P(B) = \frac{1}{4}$, then $P(B'|A)$ is equal to

(A) $\frac{1}{4}$ **(B)** $\frac{1}{3}$ **(C)** $\frac{3}{4}$ **(D)** 1 R

11. A linear programming problem is one that is concerned with

(A) finding the optimal value (maximum or minimum) of a linear function of several variables

(B) finding the limiting values of a linear function of several variables

(C) finding the lower limit of a linear function of several variables

(D) finding the upper limits of a linear function of several variables AI U+R

12. If A be origin O and $A(x, y, z)$ be any point, then $\overline{OA} =$

(A) $x\hat{i} + y\hat{j} + z\hat{k}$ **(B)** $x\hat{i} - y\hat{j} + z\hat{k}$ **(C)** $x\hat{i} + y\hat{j} - z\hat{k}$ **(D)** $x\hat{i} - y\hat{j} - z\hat{k}$ E

13. A and B play with two dice on the condition that A wins if he thrown 6 before B throw 7, then the probability that A wins is

(A) $\frac{36}{1141}$ **(B)** $\frac{30}{61}$ **(C)** $\frac{172}{1141}$ **(D)** $\frac{191}{1141}$ E

14. If $P(A \cap B) = 80\%$ and $P(B) = 95\%$, then $P(A/B)$ is equal to

(A) $\frac{16}{19}$ **(B)** $\frac{8}{9}$ **(C)** $\frac{1}{10}$ **(D)** $\frac{1}{9}$ Ap

15. Three friends – Bulbul, Ipsita and Sagarika were asked to find a particular solution of the following differential equation $\frac{d^2y}{dx^2} + y = 0$.

Shown below are their solutions.

Bulbul: $y = \sin x$

Ipsita: $y = \cos x$

Sagarika: $y = \sin x + \cos x$

Whose answer is correct ?

(A) Only Bulbul **(B)** Only Sagarika

(C) Only Bulbul and Ipsita **(D)** All Bulbul, Sagarika and Ipsita.

16. $\left|3\hat{i} + 2\hat{j} + 2\hat{k}\right| =$

(A) $\sqrt{15}$ **(B)** $\sqrt{17}$ **(C)** $\sqrt{21}$ **(D)** None of these E

17. The amount of pollution content added in air in a city due to x-diesel vehicles is given by $p(x) = 0.005x^3 + 0.02x^2 + 30x$. The marginal increase in pollution content when 3 diesel vehicles are added is

(A) 29 **(B)** 30 **(C)** 31 **(D)** 32 U

18. Can two different vectors have the same magnitude?

(A) Yes **(B)** No **(C)** Cannot be determined **(D)** None of these [R]

ASSERTION-REASON BASED QUESTIONS

In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(A) Both (A) and (R) are true and (R) is the correct explanation of (A).
(B) Both (A) and (R) are true but (R) is not the correct explanation of (A).
(C) (A) is true but (R) is false.
(D) (A) is false but (R) is true.

19. Assertion (A): $\int 3x^2(\cos x^3 + 8)dx = \sin x^3 + 8x^3 + C$

Reason (R): The above integration is solved using substitution method. [U]+[R]

20. Assertion (A): Let P be a point on the line joining the points $A(0, 5, -4)$ and $B(4, -2, 1)$. If X-coordinate of P is 3, then its Y-coordinate is $-\frac{1}{4}$.

Reason (R): The equation of line passing through two points $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$ is given by [U]+[R]

$$\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1}$$

Section-B

This section has 5 Very Short Answer questions of 2 marks each.

21. Show that $\cot^{-1}\left(\frac{1-x^2}{2x}\right) + \tan^{-1}\left(\frac{x^3-3x}{1-3x^2}\right) = -\tan^{-1} x.$

OR

Prove that $3\cos^{-1}x = \cos^{-1}[4x^3 - 3x], x \in \left[\frac{1}{2}, 1\right]$ [R]

22. Find the solution of the differential equation $\frac{dy}{dx} = x^3 e^{-2y}$. [AI] [E]

OR

What family of curves does the solution of the differential equation $2 \times \frac{dy}{dx} = 7 + y$ represent ? Show your work.

23. Find the maximum value of $\frac{\log x}{x}$. [R]

24. Find $\int \frac{x+1}{(x+2)(x+3)} dx$. [E]

25. Find $\frac{dm}{dn}$ at $(m, n) = (-1, 1)$ where: [A]

$5m^2 + 2m - n^{\frac{-2}{3}} = 0$

Section-C

There are 6 short answer questions in this section. Each is of 3 marks.

26. Evaluate the integral: $I = \int_0^{\frac{\sqrt{3}}{2}} \left[\frac{\sin^{-1} x}{(1-x^2)^{\frac{3}{2}}} dx\right]$.

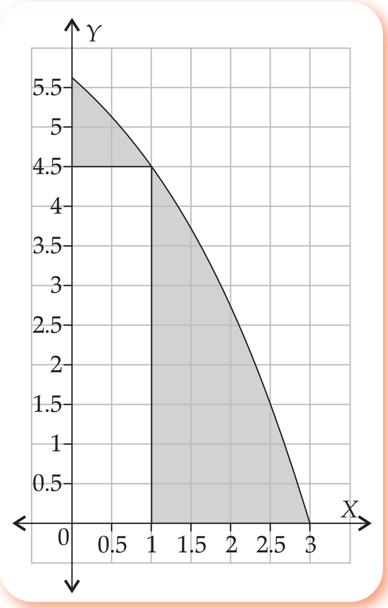
Show your work.

OR

Evaluate: $\int_0^{2\pi} \frac{dx}{1+e^{\sin x}}$. [E]

27. A bag contains $(2n + 1)$ coins. It is known that $(n - 1)$ of these coins have a head on both sides, whereas the rest of the coins are fair. A coin is picked up at random from the bag and is tossed. If the probability that the toss results in a head is $\frac{31}{42}$, determine the value of n. [A]

28. Shown below is the graph of $f(x) = \frac{-3}{8}(x+1)^2 + 6$ in the first quadrant.



(i) Find the area of the shaded region. Show your steps.

(ii) In the second quadrant, if an identical unshaded rectangle is drawn under the graph of $f(x)$, would the area of the shaded area be the same as part (i) ? Justify your answer.

OR

Find the area lying in the first quadrant and bounded by circle $x^2 + y^2 = 4$ and the lines $x = 0$ and $x = 2$. [C]

29. Find the particular solution of the differential equation

$$x\frac{dy}{dx} = y - x\tan\left(\frac{y}{x}\right)$$

(Given that $y = \frac{\pi}{4}$ at $x = 1$) [E]

30. Two helicopters flying to Kedar Hills are moving in straight lines represented by
$2\hat{i} + 3\hat{j} + 2\hat{k} + \mu(3\hat{i} + \hat{j} + 2\hat{k})$ and $\hat{i} + 2\hat{j} + 3\hat{k} + \lambda(3\hat{i} + \hat{j} + 2\hat{k})$ rjespectively.
Find the shortest possible distance between the helicopters during the flight. Show your steps and give a valid reason. [U]+[R]

31. Find the values of p and q, for which

$$f(x) = \begin{cases} \frac{1-\sin^3 x}{3\cos^2 x}, & \text{if } x < \frac{\pi}{2} \\ p, & \text{if } x = \frac{\pi}{2}, \\ \frac{q(1-\sin x)}{(\pi - 2x)^2}, & \text{if } x > \frac{\pi}{2} \end{cases}$$

is continuous at $x = \frac{\pi}{2}$. [U]+[E]

OR

If $x = a(\cos 2\theta + 2\theta \sin 2\theta)$ and $y = a(\sin 2\theta - 2\theta \cos 2\theta)$, find $\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{8}$. [U]

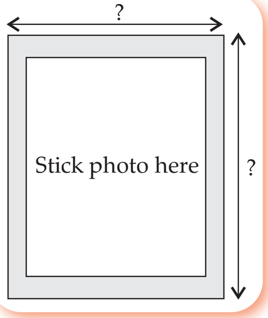
Section-D

There are 4 long answer questions in this section. Each question is of 5 marks.

32. Mr Aithal, a mathematics teacher, announces the following activity in his classroom and assures grand prizes for the winners.

Instructions:

* Make a rectangular photo frame of total area 80 cm² using a chart paper.
* The frame should have a margin of 1.25 cm each at the top and the bottom.
* The frame should have a margin of 1 cm each on the left and the right sides.
* The area available at the centre to stick the photo should be maximum.



What must be the dimensions of such a photo frame? Show your work [A]

33. Let N denote the set of all natural numbers and R be the relation on N × N defined by (a, b) R(c, d) if $ad(b + c) = bc(a + d)$. Show that R is an equivalence relation. [U]

OR

If $(\tan^{-1} x)^2 + (\cot^{-1} x)^2 = \frac{5\pi^2}{8}$, find x. [E]

34. Find the equation of line joining $P(11, 7)$ and $Q(5, 5)$ using determinants. Also, find the value of k, if $R(–1, k)$ is the point such that area of ΔPQR is 9 sq m. [A]

OR

(a) Examine the consistency of the following system of equations.

$x + 3y = 5$ and $2x + 6y = 8$

(b) Consider the matrix $X = \begin{bmatrix} p & -2 & -3 \\ -2 & 2 & 6 \\ 1 & 3 & q \end{bmatrix}$, where p and $q \in R$.

The co-factor of element 6 is (-11) and the minor of element 2 is 0.

Find the values of p and q. Show your work. [R]

35. Let $\vec{a} = \hat{i} + 4\hat{j} + 2\hat{k}$, $\vec{b} = 3\hat{i} - 2\hat{j} + 7\hat{k}$ and $\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$. Find the vector $\vec{d}$ which is perpendicular to both $\vec{a}$ and $\vec{b}$ and $\vec{c}.\vec{d} = 15$. [A]

Section-E

In this section there are 3 case study questions of 4 marks each.

36. **Case Study 1:** On her birthday, Seema decided to donate some money to children of an orphanage home. If there were 8 children less, everyone would have got ₹10 more. However, if there were 16 children more, everyone would have got ₹10 less. Let the number of children be x and the amount distributed by Seema for one child be y (in ₹). [A]

(i) Find the equations related to the given problem in terms of x and y. **[2]**

OR

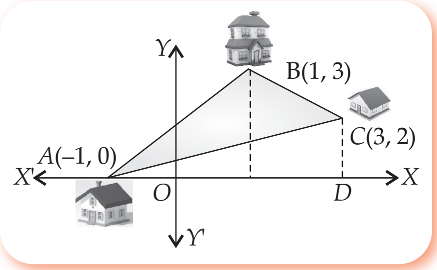
Find the number of children. How much amount is given to each child by Seema? **[2]**

(ii) Write the equations in form of matrix representation for the information given above? **[1]**

(iii) How much amount Seema spends in distributing the money to all the students of the Orphanage? **[1]**

37. Case Study 2: Read the following text and answer the following questions on the basis of the same:

Three friends Amit, Sumit and Rahul live in a society. The location of their houses in the society forms a triangular shape. The location of the three houses of that society is represented by points A(–1, 0), B(1, 3) and C(3, 2) as shown in the given figure.



(i) Find the equation of line AB. **[1]**

(ii) Find the equation of line BC. **[1]**

(iii) Find the area of region ABCD. **[2]**

OR

Evaluate: $\int_{0}^{1/2} \sqrt{1-(x-1)^2}$ **[2]**

38. Case Study 3: Shaanta is a wholesale dry fruit trader who deals with figs and cashews. The maximum capital available with her in a certain month is ₹ 24,00,000. She has a warehouse with a maximum capacity to store 50 quintals of dry fruits at a time. Figs cost her ₹ 40,000 per quintal and cashews ₹ 60,000 per quintal. She earns a profit of ₹ 2,000 per quintal on figs and ₹ 3,000 per quintal on cashews. U

(i) How many quintals of figs and cashews? **[2]**

(ii) Should she purchase that month to make a maximum profit? Show your steps. **[2]**